

Quiz 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.6 #22 (5 marks) Let $Ax = \mathbf{0}$ be a homogeneous system of n linear equations in n unknowns, and let Q be an invertible $n \times n$ matrix. Show that $Ax = \mathbf{0}$ has just the trivial solution if and only if $(QA)x = \mathbf{0}$ has just the trivial solution.

[\Rightarrow]

Premise: $Ax = \mathbf{0}$ has just the trivial solution

Conclusion: $(QA)x = \mathbf{0}$ has just the trivial solution.

Since Q is invertible $Q = E_1 E_2 \dots E_k$ where E_i are elem. matrices by TFAE. So $(QA)x = \mathbf{0}$

$E_1 E_2 \dots E_k A x = \mathbf{0}$ is the equivalent of performing k elem. row op.

Performing elem. row. op. does not change the solution set.

$\therefore QAx = \mathbf{0}$ has only the trivial solution

[\Leftarrow]

Premise: $(QA)x = \mathbf{0}$ has just the trivial solution

Conclusion: $Ax = \mathbf{0}$ has just the trivial solution
 QA is invertible by TFAE $\therefore A$ is invertible
 $\therefore Ax = \mathbf{0}$ has only the trivial solution by TFAE

Question 2. §1.7 #34 (5 marks) Find all 3×3 diagonal matrices A that satisfy $A^2 - 3A - 4I = \mathbf{0}$.

$$\text{Let } A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \text{ so } A^2 - 3A - 4I = \mathbf{0}$$

$$\begin{bmatrix} a^2 - 3a - 4 & 0 & 0 \\ 0 & b^2 - 3b - 4 & 0 \\ 0 & 0 & c^2 - 3c - 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} \text{So } a^2 - 3a - 4 &= 0 & b^2 - 3b - 4 &= 0 & c^2 - 3c - 4 &= 0 \\ (a-4)(a+1) &= 0 & (b-4)(b+1) &= 0 & (c-4)(c+1) &= 0 \\ a=4 & \quad a=-1 & b=4 & \quad b=-1 & c=4 & \quad c=-1 \end{aligned}$$

$$\therefore A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \text{ where } a, b, c \in \{4, -1\}$$