Dawson College: **Linear Algebra**: 201-NYC-05-S06: Fall 2015

Name: ____________________
Student ID: ____________________

Test 1

This test is graded out of 46 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1.

a. *(6 marks)* Solve the following system by Gauss-Jordan elimination:

$$\begin{array}{ccccccccccc} 3x_1 & + & 3x_2 & + & 7x_3 & - & 3x_4 & + & x_5 & = & 3 \\ 2x_1 & + & 3x_2 & + & 3x_3 & + & x_4 & - & 2x_5 & = & 1 \\ 4x_1 & & & + & 17x_3 & - & 2x_4 & - & x_5 & = & 1 \end{array}$$

b. *(1 mark)* Find two particular solution to the above system.

c. *(1 mark)* Find a solution to the above system where $x_3 = 1$.

Question 2. Consider the matrices:

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 0 \\ 3 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 & 1 \\ -3 & -4 & 0 \end{bmatrix} C = \begin{bmatrix} 2 & 1 \\ 0 & 2 \\ 3 & -1 \end{bmatrix} D = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} F = \begin{bmatrix} 5 & -1 \\ 0 & -2 \\ 1 & 0 \end{bmatrix}$$

a. *(2 marks)* Compute the following, if possible.

$$\text{tr}(BAF)$$

b. *(2 marks)* Compute the following, if possible.

$$CAB$$

c. *(2 marks)* Compute the following, if possible.

$$(FD^{-1})^T$$

d. *(5 marks)* Find E, if possible.

$$(I - E^T)^{-1} = (\text{tr}(D)D^2)^T$$

Question 3. Consider

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 4 & 3 & 0 \\ 3 & 2 & \frac{1}{2} \end{bmatrix}.$$

a. *(5 marks)* Find A^{-1}.

b. *(3 marks)* Solve for X where $AX = B$ and

$$B = \begin{bmatrix} 1 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & 2 & -1 \\ -4 & 2 & \frac{1}{2} & 0 & 0 \end{bmatrix}$$

Question 4. *(4 marks)* Prove: Let A be a square matrix. If $A^4 = AAAA = 4I$ then A is invertible.

Question 5. *(4 marks)* Prove: $AB = BA$ if and only if $(A+B)(A-B) = (A-B)(A+B)$

Question 6. *(5 marks)* Express

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

as a product of 4 elementary matrices.

Question 7. The augmented matrix of a linear system is given by

$$\begin{bmatrix} 1 & 2 & 3 & 4 & \pi \\ 0 & \sqrt{2} & 4 & 5 & 6 \\ 0 & 0 & 0 & a^2-1 & b^2-a^2 \end{bmatrix}$$

If possible for what values of a and b there is

a. *(2 marks)* no solution? Justify.

b. *(2 marks)* exactly one solution? Justify.

c. *(2 marks)* infinitely many solutions? Justify.

Bonus Question.*(5 marks)*
If A, B and $A+B$ are invertible matrices of the same size then show that $A^{-1}+B^{-1}$ is invertible and find a formula for $(A^{-1}+B^{-1})^{-1}$.