Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S7: Fall 2015

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Quiz 10

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §4.1 #7 (5 marks) Determine whether each set equipped with the given operations is a vector space. For those that are not vector spaces identify the vector space axiom that fail.

The set of all triples of real numbers with the standard vector addition but with scalar multiplication defined by

$k(x,y,z) = (k^2x, k^2y, k^2z)$ Lets verify the axiom: $(r+s)\vec{v} = r\vec{v} + s\vec{v}$

Let $\vec{v} = (x,y,z) \in \mathbb{R}^3$, $r,s \in \mathbb{R}$

$$\begin{aligned} LHS &= (r+s)(x,y,z) \\ &= ((r+s)^2x, (r+s)^2y, (r+s)^2z) \\ &= ((r^2+2rs+s^2)x, (r^2+2rs+s^2)y, (r^2+2rs+s^2)z) \end{aligned}$$

$$\begin{aligned} RHS &= r(x,y,z) + s(x,y,z) \\ &= (r^2x, r^2y, r^2z) + (s^2x, s^2y, s^2z) \\ &= ((r^2+s^2)x, (r^2+s^2)y, (r^2+s^2)z) \end{aligned}$$

$LHS \neq RHS$ $\therefore$ axiom does not hold

$\therefore$ not a vector space.

Question 2. §4.1 #8 *(5 marks)* Determine whether each set equipped with the given operations is a vector space. For those that are not vector spaces identify the vector space axiom that fail.

The set of all 2×2 invertible matrices with the standard matrix addition and scalar multiplication.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \in \{M \mid M \in M_{2\times2} \text{ and } |M| \neq 0\}$$

since $\det(I_2) = 1 \neq 0$ and $\det(-I_2) = 1 \neq 0$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \notin \{M \mid M \in M_{2\times2} \text{ and } |M| \neq 0\}$$

since $\det(O_2) = 0$

$\therefore$ closure under addition fails

$\therefore$ not a vector space.