Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S7: Fall 2015

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Quiz 11

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §4.2 #2f (5 marks) Use the Subspace Test to determine which of the following are subspaces of $\mathscr{M}_{n\times n}$.
The set of all $n\times n$ matrices A for which $A\mathbf{x}=\mathbf{0}$ has only the trivial solution.

$Ax=0$ has only the trivial solution iff $|A|\neq 0$.

Let $W=\{A \mid A\in M_{n\times n} \text{ and } |A|\neq 0\}$.

Lets show the subspace ~~test~~ fails. Lets show that it fails additive closure.

Let $A=I_n$ and $B=-I_n$

$\det(A)=1$ and $\det(B)=(-1)^n$

$A+B=0$ so $\det(A+B)=0$ $\therefore A+B\notin W$

Question 2. §4.2 #13 *(5 marks)* Determine whether the following polynomials span $\mathscr{P}_2$.

$p_1=1-x+2x^2,\qquad p_2=3+x,\qquad p_3=5-x+4x^2,\qquad p_4=-2-2x+2x^2$

Let $p(x)=a+bx+cx^2\in P_2$

$a+bx+cx^2 = c_1p_1+c_2p_2+c_3p_3+c_4p_4$

$a+bx+cx^2 = c_1(1-x+2x^2)+c_2(3+x)+c_3(5-x+4x^2)+c_4(-2-2x+2x^2)$

$a+bx+cx^2 = (c_1+3c_2+5c_3-2c_4)+(-c_1+c_2-c_3-2c_4)x + (2c_1+4c_3+2c_4)x^2$

$$\begin{aligned} c_1+3c_2+5c_3-2c_4&=a\\ -c_1+c_2-c_3-2c_4&=b\\ 2c_1\qquad\;+4c_3+2c_4&=c \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1&3&5&-2&a\\ -1&1&-1&-2&b\\ 2&0&4&2&c \end{array}\right]$$

$\sim$ $R_1+R_2\to R_2$, $-2R_1+R_3\to R_3$
$$\left[\begin{array}{cccc|c} 1&3&5&-2&a\\ 0&4&4&-4&a+b\\ 0&-6&-6&6&c-2a \end{array}\right]$$

$\sim$ $\frac{3}{2}R_2+R_3\to R_3$
$$\left[\begin{array}{cccc|c} 1&3&5&-2&a\\ 0&4&4&-4&a+b\\ 0&0&0&0&c-2a+\frac{3}{2}a+\frac{3}{2}b \end{array}\right]$$

Consistent iff $c-\frac{1}{2}a+\frac{3}{2}b=0$

$\therefore$ not all $p(x)$ can be written as a linear combination of p_1,p_2,p_3,p_4 $\therefore$ the set does not span P_2