

## Quiz 2

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** §1.2 #3a (3 marks) Suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given row echelon form. Solve the system.

$$\begin{bmatrix} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

Sub ④, ③ into ①

$$x_1 - 3(-9) + 4(5) = 7$$

$$x_1 = -37$$

$$\begin{aligned} \textcircled{1} \quad & x_1 - 3x_2 + 4x_3 = 7 \\ \textcircled{2} \quad & x_2 + 2x_3 = 2 \\ \textcircled{3} \quad & x_3 = 5 \end{aligned}$$

$$\therefore (x_1, x_2, x_3) = (-37, -8, 5)$$

sub ③ into ②

$$x_2 + 2(5) = 2$$

$$\textcircled{4} \quad x_2 = -8$$

**Question 2.** §1.2 #30 (4 marks) Solve the following system, where  $a, b$  and  $c$  are constants.

$$\begin{aligned} x_1 + x_2 + x_3 &= a \\ 2x_1 + 2x_3 &= b \\ 3x_2 + 3x_3 &= c \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 1 & a \\ 2 & 0 & 2 & b \\ 0 & 3 & 3 & c \end{bmatrix}$$

$$\sim -2R_1 + R_2 \rightarrow R_2 \begin{bmatrix} 1 & 1 & 1 & a \\ 0 & -2 & 0 & b-2a \\ 0 & 3 & 3 & c \end{bmatrix}$$

$$\sim 2R_3 \rightarrow R_3 \begin{bmatrix} 1 & 1 & 1 & a \\ 0 & -2 & 0 & b-2a \\ 0 & 6 & 6 & 2c \end{bmatrix}$$

$$\sim 6R_1 \rightarrow R_1 \begin{bmatrix} 6 & 6 & 6 & 6a \\ 0 & -2 & 0 & b-2a \\ 0 & 0 & 6 & 2c+3b-6a \end{bmatrix}$$

$$\sim -R_3 + R_1 \rightarrow R_1 \begin{bmatrix} 6 & 6 & 0 & 12a-3b-2c \\ 0 & -2 & 0 & b-2a \\ 0 & 0 & 6 & 2c+3b-6a \end{bmatrix}$$

**Question 3.** §1.2 #31 (3 marks) Find two different row echelon forms of

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

This exercise shows that a matrix can have multiple row echelon forms.

$$\sim 3R_2 + R_1 \rightarrow R_1 \begin{bmatrix} 6 & 0 & 0 & 6a-2c \\ 0 & -2 & 0 & b-2a \\ 0 & 0 & 6 & 2c+3b-6a \end{bmatrix}$$

$$\therefore (x_1, x_2, x_3) = (a - \frac{1}{3}c, a - \frac{1}{2}b, \frac{1}{2}b - a + \frac{1}{3}c)$$

$$\sim \begin{aligned} & \frac{1}{6}R_1 \rightarrow R_1 \\ & -\frac{1}{2}R_2 \rightarrow R_2 \\ & \frac{1}{6}R_3 \rightarrow R_3 \end{aligned} \begin{bmatrix} 1 & 0 & 0 & a - \frac{1}{3}c \\ 0 & 1 & 0 & a - \frac{1}{2}b \\ 0 & 0 & 1 & \frac{1}{2}b - a + \frac{1}{3}c \end{bmatrix}$$

$$\textcircled{1} \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \sim -2R_1 + R_2 \rightarrow R_2 \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$\begin{aligned} \textcircled{2} \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} &\sim R_1 \leftrightarrow R_2 \begin{bmatrix} 2 & 7 \\ 1 & 3 \end{bmatrix} \\ &\sim 2R_2 \rightarrow R_2 \begin{bmatrix} 2 & 7 \\ 2 & 6 \end{bmatrix} \\ &\sim -R_1 + R_2 \rightarrow R_2 \begin{bmatrix} 2 & 7 \\ 0 & -1 \end{bmatrix} \\ &\sim \begin{array}{l} \frac{1}{2}R_1 \rightarrow R_1 \\ -R_2 \rightarrow R_2 \end{array} \begin{bmatrix} 1 & \frac{7}{2} \\ 0 & 1 \end{bmatrix} \end{aligned}$$