

Quiz 3

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.2 #5i (3 marks) Consider the matrices

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}, E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

Compute the given expression (if possible): $\text{tr}(DD^T)$.

$$\begin{aligned} \text{tr}(DD^T) &= \text{tr} \left(\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix} \right) = \text{tr} \left(\begin{bmatrix} 30 & 1 & 21 \\ 1 & 2 & 1 \\ 21 & 1 & 29 \end{bmatrix} \right) \\ &= 30 + 2 + 29 = 61 \end{aligned}$$

Question 2. §1.3 #20b (2 marks) Show that if A is an $m \times n$ matrix and $A(BA)$ is defined, then B is an $n \times m$ matrix.

In order for the product BA to be defined B needs the same amount of columns as A has rows. So m columns.

In order for the product $A(BA)$ to be defined BA needs the same amount of rows as A has columns. So BA needs to have n rows. So B needs to have n rows.

Similarly

Question 3. §1.4 #16 (3 marks) Use the given information to find A . $\begin{matrix} m \times n & n \times m & m \times n \end{matrix}$

$$(5A^T)^{-1} = \begin{bmatrix} -3 & -1 \\ 5 & 2 \end{bmatrix}$$

$$\left[(5A^T)^{-1} \right]^{-1} = \begin{bmatrix} -3 & -1 \\ 5 & 2 \end{bmatrix}^{-1}$$

$$5A^T = \frac{1}{(-3)(2) - (-1)(5)} \begin{bmatrix} 2 & 1 \\ -5 & -3 \end{bmatrix}$$

$$5A^T = -1 \begin{bmatrix} 2 & 1 \\ -5 & -3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} -2/5 & -1/5 \\ 1 & 3/5 \end{bmatrix}$$

$$A = \begin{bmatrix} -2/5 & 1 \\ -1/5 & 3/5 \end{bmatrix}$$

Question 4. §1.4 #TF (2 marks) Determine whether the statement is true or false, and justify your answer.

Two $n \times n$ matrices, A and B , are inverses of one another if and only if $AB = BA = 0$.

False A and B are inverses of each other iff $AB = BA = I$.