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Quiz 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.5 #41 *(5 marks)* Prove that if A and B are $m \times n$ matrices, then A and B are row equivalent if and only if A and B have the same reduced row echelon form.

[$\Rightarrow$] premise: A and B are row equivalent
$\therefore \exists E_i$ s.t. $E_k \cdots E_2 E_1 A = B$ where E_i are elem. matrices

conclusion: A and B have the same R.R.E.F. R.

Perform Gauss Jordan on B, $B \sim \ell$ elem. row. op. $\sim R$
From the ℓ elem. row. op. $\exists F_i$ s.t. $F_\ell \cdots F_2 F_1 B = R$
So $F_\ell \cdots F_2 F_1 E_k \cdots E_2 E_1 A = R$ $\therefore$ A and B have the same R.R.E.F.

[$\Leftarrow$] premise: A and B have the same R.R.E.F. R.

conclusion: A and B are row equivalent

Perform Gauss-Jordan on A and B, $A \sim k$ elem. row. op. $\sim R$
From the k elem. row. op. $\exists E_i$ s.t. $E_k \cdots E_2 E_1 A = R$
$B \sim \ell$ elem. row op $\sim R$. From the ℓ elem. row. op. $\exists F_i$ s.t. $F_\ell \cdots F_2 F_1 B = R$ So $F_\ell \cdots F_2 F_1 B = E_k \cdots E_2 E_1 A$, so $A = E_1^{-1} \cdots E_k^{-1} F_\ell \cdots F_2 F_1 B$ $\therefore$ A and B are row equiv.

Question 2. §1.6 #5 *(2 marks)* Solve the system by inverting the coefficient matrix.

$$\begin{aligned} x + y + z &= 5 \\ x + y - 4z &= 10 \\ -4x + y + z &= 0 \end{aligned}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & -4 & 0 & 1 & 0 \\ -4 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \begin{array}{l} -R_1+R_2 \to R_2 \\ 4R_1+R_3 \to R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -5 & -1 & 1 & 0 \\ 0 & 5 & 5 & 4 & 0 & 1 \end{array}\right]$$

$$\sim R_2 \leftrightarrow R_3 \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 5 & 5 & 4 & 0 & 1 \\ 0 & 0 & -5 & -1 & 1 & 0 \end{array}\right]$$

$$\sim \begin{array}{l} R_3+R_2 \to R_2 \\ \frac{-1}{5}R_3 \to R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 5 & 0 & 3 & 1 & 1 \\ 0 & 0 & 1 & 1/5 & -1/5 & 0 \end{array}\right]$$

$$\sim \begin{array}{l} -R_3+R_1 \to R_1 \\ \frac{1}{5}R_2 \to R_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 4/5 & 1/5 & 0 \\ 0 & 1 & 0 & 3/5 & 1/5 & 1/5 \\ 0 & 0 & 1 & 1/5 & -1/5 & 0 \end{array}\right]$$

$$\sim -R_2+R_1 \to R_1 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/5 & 0 & -1/5 \\ 0 & 1 & 0 & 3/5 & 1/5 & 1/5 \\ 0 & 0 & 1 & 1/5 & -1/5 & 0 \end{array}\right]$$

$$x = A^{-1}b = \begin{bmatrix} 1/5 & 0 & -1/5 \\ 3/5 & 1/5 & 1/5 \\ 1/5 & -1/5 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 10 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}$$