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Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §2.1 #41 *(5 marks)* Show that the value of the following determinant is independent of θ.

$$\begin{vmatrix} \sin\theta & \cos\theta & 0 \\ -\cos\theta & \sin\theta & 0 \\ \sin\theta-\cos\theta & \sin\theta+\cos\theta & 1 \end{vmatrix} = a_{13}C_{13} + a_{23}C_{23} + a_{33}C_{33}$$

$$= 0\,C_{13} + 0\,C_{23} + 1(-1)^{3+3}M_{33}$$

$$= \begin{vmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{vmatrix} = \sin^2\theta + \cos^2\theta = 1$$

Question 2. §2.2 #16 *(5 marks)* Evaluate the determinant of the given matrix by reducing the matrix to row echelon form (or triangular form).

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ -\frac{1}{3} & \frac{2}{3} & 0 & 0 \end{bmatrix}$$

$$\sim \begin{array}{c} 2R_2 \to R_2 \\ 3R_3 \to R_3 \\ 3R_4 \to R_4 \end{array} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 2 & 1 & 1 & 0 \\ -1 & 2 & 0 & 0 \end{bmatrix}$$

$$\sim R_1 \leftrightarrow R_2 \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 2 & 1 & 1 & 0 \\ -1 & 2 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{array}{c} -2R_1 + R_3 \to R_3 \\ R_1 + R_4 \to R_4 \end{array} \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & -1 & -3 & -2 \\ 0 & 3 & 2 & 1 \end{bmatrix}$$

$$\sim \begin{array}{c} R_2 + R_3 \to R_3 \\ -3R_2 + R_4 \to R_4 \end{array} \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -2 & -1 \\ 0 & 0 & -1 & -2 \end{bmatrix}$$

$$\sim -\tfrac{1}{2}R_3 + R_4 \to R_4 \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -2 & -1 \\ 0 & 0 & 0 & -\frac{3}{2} \end{bmatrix} = B$$

$$(2)(3)(3)(-1)\,\det A = \det B$$
$$-18\det A = 1(1)(-2)\left(-\tfrac{3}{2}\right)$$
$$\det A = -\tfrac{1}{6}$$