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Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §2.3 #41 *(5 marks)* Prove that a square matrix A is invertible if and only if A^TA is invertible.

[=>] Premise: A is invertible
Conclusion: A^TA is invertible
A invertible => $\det A \neq 0$, So $\det A^TA = \det A^T \det A = \det A \det A \neq 0$
$\neq 0$ $\neq 0$

$\therefore A^TA$ is invertible

[<=] Premise: A^TA is invertible
Conclusion: A is invertible
A^TA invertible => $\det A^TA \neq 0$
$\det A^T \det A \neq 0$
$(\det A)^2 \neq 0$
$\Rightarrow \det A \neq 0$

Question 2. §3.1 #27 *(5 marks)* Find all scalars c_1, c_2 and c_3 such that

$$c_1(1,-1,0)+c_2(3,2,1)+c_3(0,1,4)=(-1,1,19)$$

$$\begin{aligned} C_1 + 3C_2 &= -1 \\ -C_1 + 2C_2 + C_3 &= 1 \\ 2C_2 + 4C_3 &= 19 \end{aligned}$$

$$\begin{bmatrix} 1 & 3 & 0 & -1 \\ -1 & 2 & 1 & 1 \\ 0 & 1 & 4 & 19 \end{bmatrix} \sim R_1+R_2 \to R_2 \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 5 & 1 & 0 \\ 0 & 1 & 4 & 19 \end{bmatrix} \sim 5R_3 \to R_3 \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 5 & 1 & 0 \\ 0 & 5 & 20 & 95 \end{bmatrix}$$

$$\sim -R_2+R_3 \to R_3 \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 5 & 1 & 0 \\ 0 & 0 & 19 & 95 \end{bmatrix}$$

$$\sim \frac{1}{19}R_3 \to R_3 \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 5 & 1 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$\sim -R_3+R_2 \to R_2 \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 5 & 0 & -5 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$\sim \frac{1}{5}R_2 \to R_2 \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$\sim -3R_2+R_1 \to R_1 \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$\therefore (C_1, C_2, C_3) = (2, -1, 5)$.