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Quiz 8

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. 5.19c (5 marks) For each pair of lines, determine whether the lines intersect each other. If so, determine the point of intersection.

$\mathbf{x} = (0,4,2) + t(-1,1,1)$ and $\mathbf{x} = (-4,-2,9) + s(1,1,-2)$

$-t = -4+s \iff t = 4-s$ ①
$4+t = -2+s$ ②
$2+t = 9-2s$ ③

sub ① into ②
$4+(4-s) = -2+s$
$10 = 2s$
$5 = s$

sub into ①
$t = 4-5$
$= -1$

sub s, t into ③
$2+(-1) \stackrel{?}{=} 9-2(5)$
$1 \neq -1$
∴ not consistent
∴ the two lines do not intersect

Question 2. §3.4 #19 *(5 marks)* Find the general solution to the linear system and confirm that the row vectors of the coefficient matrix are orthogonal to the solution vectors.

$x_1 + 5x_2 + x_3 + 2x_4 - x_5 = 0$
$x_1 - 2x_2 - x_3 + 3x_4 + 2x_5 = 0$

$$\begin{bmatrix} 1 & 5 & 1 & 2 & -1 & 0 \\ 1 & -2 & -1 & 3 & 2 & 0 \end{bmatrix}$$

$$\sim -R_1+R_2 \to R_2 \begin{bmatrix} 1 & 5 & 1 & 2 & -1 & 0 \\ 0 & -7 & -2 & 1 & 3 & 0 \end{bmatrix}$$

$$\sim -\tfrac{1}{7}R_2 \to R_2 \begin{bmatrix} 1 & 5 & 1 & 2 & -1 & 0 \\ 0 & 1 & 3/7 & -1/7 & -3/7 & 0 \end{bmatrix}$$

$$\sim -5R_2+R_1 \to R_1 \begin{bmatrix} 1 & 0 & -3/7 & 19/7 & 8/7 & 0 \\ 0 & 1 & 2/7 & -1/7 & -3/7 & 0 \end{bmatrix}$$

$x_3 = s$
$x_4 = t$ $\quad s,t,u \in \mathbb{R}$
$x_5 = u$
$x_1 = \frac{3}{7}s - \frac{19}{7}t - \frac{8}{7}u$
$x_2 = -\frac{2}{7}s + \frac{1}{7}t + \frac{3}{7}u$

Let $a_1 = (1,5,1,2,-1)$
$a_2 = (1,-2,-1,3,2)$

$a_1 \cdot \mathbf{x}$
$= (1,5,1,2,-1) \cdot (\frac{3}{7}s - \frac{19}{7}t - \frac{8}{7}u, -\frac{2}{7}s + \frac{1}{7}t + \frac{3}{7}u, s, t, u)$
$= \frac{3}{7}s - \frac{19}{7}t - \frac{8}{7}u - \frac{10}{7}s + \frac{5}{7}t + \frac{15}{7}u + s + 2t - u$
$= 0$ ∴ $a_1 \perp \mathbf{x}$

$a_2 \cdot \mathbf{x} = (1,-2,-1,3,2) \cdot (\frac{3}{7}s - \frac{19}{7}t - \frac{8}{7}u, -\frac{2}{7}s + \frac{1}{7}t + \frac{3}{7}u, s, t, u)$
$= \frac{3}{7}s - \frac{19}{7}t - \frac{8}{7}u + \frac{4}{7}s - \frac{2}{7}t - \frac{6}{7}u - s + 3t + 2u$
$= 0$
∴ $a_2 \perp \mathbf{x}$