**Dawson College**: **Linear Algebra**: 201-NYC-05-S07: Fall 2015

**Name:** ____________________
**Student ID:** ____________________

# Test 1

This test is graded out of 46 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.**

a. *(6 marks)* Solve the following system by Gauss-Jordan elimination:

$$\begin{array}{rcrcrcrcl} 3x_1 & + & 3x_2 & + & 7x_3 & - & 3x_4 & = & 0 \\ 2x_1 & + & 3x_2 & + & 3x_3 & + & x_4 & = & 0 \\ 4x_1 & & & + & 17x_3 & - & 2x_4 & = & 0 \\ 9x_1 & + & 6x_2 & + & 27x_3 & - & 4x_4 & = & 0 \end{array}$$

b. *(1 mark)* Find two particular nontrivial solution to the above system.

c. *(1 mark)* Find a solution to the above system where $x_1 = 1$.

**Question 2.** Consider the matrices:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -2 & 0 & 3 \\ 0 & 3 & 0 \end{bmatrix},\ B = \begin{bmatrix} 2 & 1 & 0 \\ -3 & 4 & 0 \end{bmatrix}\ C = \begin{bmatrix} 2 & -1 \\ 1 & 2 \\ 0 & 1 \end{bmatrix}\ D = \begin{bmatrix} 2 & -2 \\ 1 & -2 \end{bmatrix}\ F = \begin{bmatrix} 2 & -1 \\ 3 & -2 \\ 1 & 0 \end{bmatrix}$$

a. *(2 marks)* Compute the following, if possible.

$$BIF$$

b. *(2 marks)* Compute the following, if possible.

$$ACAB$$

c. *(2 marks)* Compute the following, if possible.

$$F(D^{-1})^T B$$

d. *(5 marks)* Determine $E$, if possible.

$$DED^T = \operatorname{tr}(BIF)BF$$

**Question 3.** Consider

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 4 & 9 & 0 \\ 3 & 2 & 1 \end{bmatrix}.$$

a. *(4 marks)* Find $A^{-1}$.

b. *(4 marks)* Determine $X$, if possible.

$$(I + AX)^{-1} = \text{tr(A)}I$$

**Question 4.** *(4 marks)* Prove: If $AB$ and $BA$ are both invertible then $A$ and $B$ are both invertible.

**Question 5.** *(4 marks)* A square matrix $A$ is called *idempotent* if $A^2 = A$. Prove: If $A$ is idempotent then $A + AB - ABA$ is idempotent for any square matrix $B$ with the same dimension as $A$.

**Question 6.** Let

$$A = \begin{bmatrix} 1 & -1 & 3 & 2 \\ 0 & 1 & 4 & 1 \\ 1 & 0 & 8 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 & 4 & 5 \\ -2 & 1 & -11 & -8 \\ -1 & 2 & 2 & 2 \end{bmatrix}$$

a. *(5 marks)* Show that $A$ and $B$ have the same reduced row echelon matrix.

b. *(3 marks)* Justify using a. that $A$ and $B$ are row equivalent.

**Question 7.** *(3 marks)* The augmented matrix of a linear system is given by

$$\begin{bmatrix} 1 & 3 & 1 & -4 & b_1 \\ 3 & -2 & 4 & 5 & b_2 \\ 4 & 1 & 5 & 1 & b_3 \\ 7 & -1 & 9 & 6 & b_4 \end{bmatrix}$$

Determine the restrictions on the $b_i$'s for the system to be consistent.

**Bonus Question.**
Given

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 2 & 0 & 2 \end{bmatrix}$$

where the entries of the matrix are elements of $\mathbb{Z}_3$. Operations on the elements of $\mathbb{Z}_3$ can be defined by the following Cayley tables:

| + | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 1 | 2 |
| 1 | 1 | 2 | 0 |
| 2 | 2 | 0 | 1 |

| × | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 2 |
| 2 | 0 | 2 | 1 |

a. *(2 marks)* Find the inverse of $A$, if possible.

b. *(3 marks)* Find all the solutions of

$$A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

where $x_i \in \mathbb{Z}_3$ and the entries of the constant matrix are elements of $\mathbb{Z}_3$.