Dawson College: **Linear Algebra**: 201-NYC-05-S07: Fall 2015

Name: ____________________
Student ID: ____________________

Test 3

This test is graded out of 40 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Given

$$\begin{array}{llll} \mathscr{L}_1: & (x,y,z) & = & (t, 1-t, -1+t) & t \in \mathbb{R} \\ \mathscr{L}_2: & (x,y,z) & = & (1-2t, t, -1+2t) & t \in \mathbb{R} \end{array}$$

a. *(7 marks)* Find the point on each line which is closest to the other.

b. *(2 marks)* Find the distance between the two lines.

Question 2. *(3 marks)* Prove or disprove: If $\mathbf{x}_1$ and $\mathbf{x}_2$ are two solutions of the nonhomogeneuous linear system $A\mathbf{x} = \mathbf{b}$, then $\mathbf{x}_1 - \mathbf{x}_2$ is a solution of the corresponding homogeneuous linear system $A\mathbf{x} = 0$.

Question 3. *(4 marks)* Determine the area of the triangle generated by the vectors $\vec{u}$ and $\vec{v}$ if $\vec{u} \cdot (\vec{v} \times (\vec{u} \times \vec{v})) = 1936$

Question 4. Let $\vec{u} = (1, \lambda, -\lambda)$, $\vec{v} = (-2\lambda, -2, 2\lambda)$ and $\vec{w} = (\lambda - 2, -5\lambda - 2, -2)$.

a. *(2 marks)* For what value(s) of λ will $\{\vec{u}, \vec{v}\}$ be linearly dependent.

b. *(3 marks)* For what value(s) of λ will $\{\vec{u}, \vec{v}, \vec{w}\}$ be linearly independent.

Question 5.[1] Let $\vec{u}$, $\vec{v}$, $\vec{p}$, and $\vec{q}$ be non-zero vectors in $\mathbb{R}^3$ and suppose $\text{span}(\{\vec{u}, \vec{v}\}) = \text{span}(\{\vec{p}, \vec{q}\})$

a. *(2 marks)* If $\text{span}(\{\vec{u}, \vec{v}\})$ is a line explain why $(\vec{u} \times \vec{v}) \times (\vec{p} \times \vec{q}) = \vec{0}$.

b. *(2 marks)* If $\text{span}(\{\vec{u}, \vec{v}\})$ is a plane explain why $(\vec{u} \times \vec{v}) \times (\vec{p} \times \vec{q}) = \vec{0}$.

[1] From a John Abbott final examination

Question 6. Let $V = \{A \mid A \in \mathscr{M}_{3\times 3}$ and A has a RREF with 2 or less leading ones$\}$.

a. *(2 marks)* Does V satisfy closure under vector addition? Justify.

b. *(2 marks)* Does V contain a zero vector? If so find it. Justify.

c. *(2 marks)* Does V satisfy closure under scalar multiplication? Justify.

d. *(2 marks)* Is V a subspace of $\mathscr{M}_{3\times 3}$? Justify.

Question 7. *(7 marks)* $W = \{p(x) = a_0 + a_1x + a_2x^2 + a_3x^3 \mid p(1) = 0 \text{ and } p(-1) = 0\}$ a subspace of P_3. Find a basis B for W. Find the coordinate vector of $p(x) = -1 - x + x^2 + x^3$ relative to the basis B.

Bonus Question. *(5 marks)*
Given

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$

where the entries of the matrix are elements of $\mathbb{Z}_3$. Operations on the elements of $\mathbb{Z}_3$ can be defined by the following Cayley tables:

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

×	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

List all the vectors of the vector space $V = \{x \mid x \in {\mathbb{Z}_3}^3 \text{ and } Ax = 0\}$ over $\mathbb{Z}_3$.