Binomial Distribution and the Poisson Distribution

Given n independent Bernoulli trials with probability of success p and of failure $q = 1-p$. The random variable X is called the binomial random variable. The binomial distribution defined by the following probability function.

$$P(X=x) = \binom{n}{x} p^x q^{n-x}$$

$$= \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

For the binomial distribution we have $\mu = np$, so $p = \frac{\mu}{n}$ and $q = 1 - \frac{\mu}{n}$. From the above we get

$$= \frac{n!}{x!(n-x)!} \left(\frac{\mu}{n}\right)^x \left(1 - \frac{\mu}{n}\right)^{n-x}$$

$$= \frac{n(n-1)\cdots(n-x+1)}{n^x} \frac{\mu^x}{x!} \frac{(1-\mu/n)^n}{(1-\mu/n)^x}$$

as $n \to \infty$ $\quad \left(1 - \frac{\mu}{n}\right)^n \to e^{-\mu}$

as $n \to \infty$ $\quad \left(1 - \frac{\mu}{n}\right)^x \to 1$

as $n \to \infty$ $\quad \frac{n(n-1)\cdots(n-x+1)}{n^x} \to 1$

So if n is large and p is small.

$$P(X=x) \approx \frac{e^{-\mu} \mu^x}{x!}$$