

Student Name: \_\_\_\_\_  
Student ID: \_\_\_\_\_

# Comprehensive Examination (CE): Logic (Oral Examination)

**Text:** Proofs and Concepts: the fundamentals of abstract mathematics by Dave Witte Morris and Joy Morris.

<http://people.uleth.ca/~dave.morris/books/proofs+concepts.pdf>

**Reading:** Chapters 1 to 3.

**Evaluation:** The CE is associated to the Probability and Statistics science course (201-BZS-05). Note that if the student fails the CE, the student cannot graduate. The CE mark for 201-BZS-05 will be either a pass or a fail. To pass the CE the student must obtain 60% or more on the oral examination.

**Sample Oral Examination:** See back of page.

**Oral Examination Date and Location:**

**Consequence to Missing the Oral Examination Date:** Failure of the CE unless a valid medical note is provided.

I hereby acknowledge that I have read, understand and agree to the terms of this Comprehensive Evaluation(CE).

**Student Signature:** \_\_\_\_\_

**Date:** \_\_\_\_\_

### Sample Oral Examination:

**Question 1.** Given the following symbolization key:

$A$ : Alexander Berkman loves Emma Goldman

$B_1$ : Alexander Berkman buys bread.

$B_2$ : Emma Goldman buys bread.

$E$ : Emma Goldman loves Alexander Berkman.

$F_1$ : Alexander Berkman buys flowers.

$F_2$ : Emma Goldman buys flowers.

$P_1$ : Alexander Berkman protests.

$P_2$ : Emma Goldman protests.

Translate each English language statement into Propositional Logic.

- a. (1 mark) Emma buys flowers and Alexander buys bread if, neither Alexander loves Emma nor Emma loves Alexander.

Translate each Propositional Logic statement into English.

- b. (1 mark)  $(\neg P_2 \wedge B_2) \iff E$

**Question 2.** (2 marks) Determine whether the following statement is a tautology, contradiction, or contingent statement. Justify your conclusion.

$$[(\neg A \rightarrow B) \wedge \neg B] \rightarrow A$$

**Question 3.** (2 marks) Determine whether the following is a valid argument. Justify your conclusion.

$$(\neg P_2 \wedge B_2) \iff E \therefore E$$

**Question 4.** (3 marks) Is the following possible? If it is possible, give an example. If it is not possible, explain why.

An invalid argument, the conclusion of which is a tautology.

**Question 5.**

- a. (0.5 mark) Translate the English statement into a propositional logic statement:  
Emma Goldman does not love Alexander Berkman if Alexander does not buy flowers.
- b. (0.5 mark) Rewrite the propositional logic statement of part a. into a logically equivalent statement using the logical connective 'or'.
- c. (0.5 mark) Give the logical negation of the statement of part b. and distribute the negation using De Morgan Laws.
- d. (0.5 mark) Translate the propositional logic statement of part c. into an English statement.

**Question 6.** (2 marks) Using a truth table: determine whether the following two statements are logically equivalent. Justify.

$$\neg A \rightarrow B$$

and

$$(\neg A \rightarrow \neg B) \iff B$$

**Question 7.** (10 marks) Using only the rules of inference and the rules of replacement show that the following argument is valid using Fitch style natural deduction:

$$P \rightarrow Q, \neg P \rightarrow R, (Q \vee R) \rightarrow S, \therefore S$$

# Rules of Inference

<b><math>\rightarrow</math>-elimination (or <math>\rightarrow</math>E)</b> (or <i>Modus ponens</i> )	$\Phi \rightarrow \Psi, \Phi \therefore \Psi$
<b>Modus tollens (or MT)</b>	$\Phi \rightarrow \Psi, \neg\Psi \therefore \neg\Phi$
<b><math>\leftrightarrow</math>-introduction (or <math>\leftrightarrow</math>I)</b> (or <i>Biconditional introduction</i> )	$\Phi \rightarrow \Psi, \Psi \rightarrow \Phi \therefore \Phi \leftrightarrow \Psi$
<b><math>\leftrightarrow</math>-elimination (or <math>\leftrightarrow</math>E)</b> (or <i>Biconditional elimination</i> )	$\Phi \leftrightarrow \Psi \therefore \Phi \rightarrow \Psi$ and $\Phi \leftrightarrow \Psi \therefore \Psi \rightarrow \Phi$
<b><math>\wedge</math>-introduction (or <math>\wedge</math>I)</b> (or <i>Conjunction introduction</i> )	$\Phi, \Psi \therefore \Phi \wedge \Psi$
<b><math>\wedge</math>-elimination (or <math>\wedge</math>E)</b> (or <i>Simplification</i> )	$\Phi \wedge \Psi \therefore \Phi$ and $\Phi \wedge \Psi \therefore \Psi$
<b><math>\vee</math>-introduction (or <math>\vee</math>I)</b> (or <i>Disjunction introduction, Addition</i> )	$\Phi \therefore \Phi \vee \Psi$
<b>Disjunction elimination (or DE)</b>	$\Phi \rightarrow \Psi, \Theta \rightarrow \Psi, \Phi \vee \Theta \therefore \Psi$
<b><math>\vee</math>-elimination (or <math>\vee</math>E)</b> (or <i>Disjunctive syllogism</i> )	$\Phi \vee \Psi, \neg\Phi \therefore \Psi$
<b>Hypothetical syllogism (or HS)</b>	$\Phi \rightarrow \Psi, \Psi \rightarrow \Theta \therefore \Phi \rightarrow \Theta$
<b>Constructive dilemma (or CD)</b>	$\Phi \rightarrow \Psi, \Theta \rightarrow \Pi, \Phi \vee \Theta \therefore \Psi \vee \Pi$
<b>Destructive dilemma (or DD)</b>	$\Phi \rightarrow \Psi, \Theta \rightarrow \Pi, \neg\Psi \vee \neg\Pi \therefore \neg\Phi \vee \neg\Theta$
<b>Absorption (or ABS)</b>	$\Phi \rightarrow \Psi \therefore \Phi \rightarrow \Phi \wedge \Psi$

# Rules of Replacement

<b>Associativity (or Asso.)</b>	$\Phi \square (\Psi \square \Theta) \equiv (\Phi \square \Psi) \square \Theta$ where $\square \in \{\wedge, \vee, \leftrightarrow\}$
<b>Commutativity (or Comm.)</b>	$\Phi \square \Psi \equiv \Psi \square \Phi$ where $\square \in \{\wedge, \vee, \leftrightarrow\}$
<b>Distributivity (or Dist.)</b>	$\Phi \wedge (\Psi \vee \Theta) \equiv (\Phi \wedge \Psi) \vee (\Phi \wedge \Theta)$ and $\Phi \vee (\Psi \wedge \Theta) \equiv (\Phi \vee \Psi) \wedge (\Phi \vee \Theta)$
<b>Double negation (or DN)</b>	$\neg\neg\Phi \equiv \Phi$
<b>De Morgan's laws (or DM)</b>	$\neg(\Phi \vee \Psi) \equiv \neg\Phi \wedge \neg\Psi$ and $\neg(\Phi \wedge \Psi) \equiv \neg\Phi \vee \neg\Psi$
<b>Transposition (or Trans.)</b>	$\Phi \rightarrow \Psi \equiv \neg\Psi \rightarrow \neg\Phi$
<b>Material implication (or MI)</b>	$\Phi \rightarrow \Psi \equiv \neg\Phi \vee \Psi$
<b>Biconditional implication (or BI)</b>	$\Phi \leftrightarrow \Psi \equiv (\Phi \rightarrow \Psi) \wedge (\Psi \rightarrow \Phi)$
<b>Exportation (or Expo.)</b>	$(\Phi \wedge \Psi) \rightarrow \Theta \equiv \Phi \rightarrow (\Psi \rightarrow \Theta)$
<b>Tautology (or Taut.)</b>	$\Phi \square \Phi \equiv \Phi$ where $\square \in \{\wedge, \vee\}$