

## Quiz 11

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

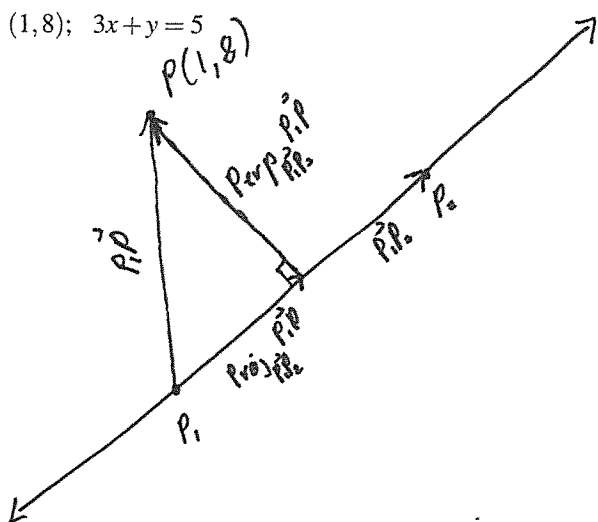
**Question 1.** §3.4 #5 (2 marks) Use the given equation of a line to find a point on the line and a vector parallel to the line.

$$\vec{x} = (3 - 5t, -6 - t) = (3, -6) + t(-5, -1) = P_0 + t\vec{d}$$

where  $P_0 = (3, -6)$  and  $\vec{d} = (-5, -1)$

**Question 2.** §3.3 #32 (4 marks) Using projections find the distance between the point and the line.

(1, 8);  $3x + y = 5$

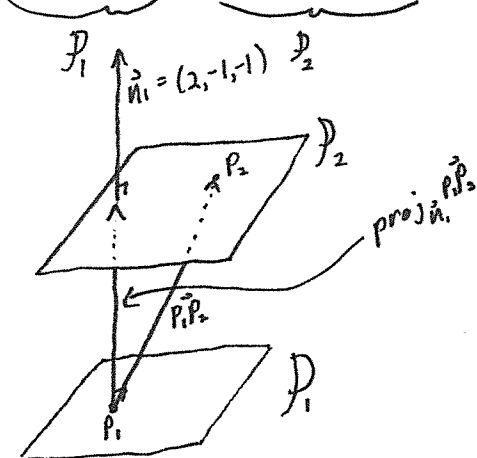


$$\begin{aligned} \vec{P_1P} &= P - P_1 = (1, 8) - (1, 2) = (0, 6) \\ \vec{P_1P_2} &= P_2 - P_1 = (2, -1) - (1, 2) = (1, -3) \\ \text{Perp}_{\vec{P_1P_2}} \vec{P_1P} &= \vec{P_1P} - \text{proj}_{\vec{P_1P_2}} \vec{P_1P} \\ &= (0, 6) - \frac{\vec{P_1P} \cdot \vec{P_1P_2}}{\vec{P_1P_2} \cdot \vec{P_1P_2}} \vec{P_1P_2} \\ &= (0, 6) - \frac{(0, 6) \cdot (1, -3)}{(1, -3) \cdot (1, -3)} (1, -3) \\ &= (0, 6) - \frac{-18}{10} (1, -3) \\ &= (0, 6) + \frac{9}{5} (1, -3) = \left(\frac{9}{5}, \frac{3}{5}\right) = \frac{3}{5} (3, 1) \\ d &= \|\text{perp}_{\vec{P_1P_2}} \vec{P_1P}\| = \left\| \frac{3}{5} (3, 1) \right\| = \frac{3}{5} \sqrt{3^2 + 1^2} = \frac{3}{5} \sqrt{10} \end{aligned}$$

Let's find 2 points on the line  
 Let  $x=1 \rightarrow 3(1)+y=5 \Leftrightarrow y=2 \therefore P_1(1, 2)$   
 Let  $x=2 \rightarrow 3(2)+y=5 \Leftrightarrow y=-1 \therefore P_2(2, -1)$

**Question 3.** §3.3 #37 (4 marks) Find the distance between the given parallel planes.

$2x - y - z = 5$  and  $-4x + 2y + 2z = 12$



$$\begin{aligned} \vec{P_1P_2} &= P_2 - P_1 \\ &= (0, 6, 0) - (0, 0, 5) \\ &= (0, 6, 5) \\ \text{proj}_{\vec{n_1}} \vec{P_1P_2} &= \frac{\vec{P_1P_2} \cdot \vec{n_1}}{\vec{n_1} \cdot \vec{n_1}} \vec{n_1} \\ &= \frac{(0, 6, 5) \cdot (2, -1, -1)}{(2, -1, -1) \cdot (2, -1, -1)} (2, -1, -1) \\ &= \frac{-11}{4+1+1} (2, -1, -1) \\ &= \frac{-11}{6} (2, -1, -1) \end{aligned}$$

Let's find a point on  $\mathcal{P}_1$ : Let  $x=y=0$   
 $2(0) - 0 - z = 5 \Rightarrow z = -5 \therefore P_1(0, 0, -5)$   
 Let's find a point on  $\mathcal{P}_2$ : Let  $x=z=0$   
 $-4(0) + 2y + 2(0) = 12 \Rightarrow y = 6 \therefore P_2(0, 6, 0)$

$$\begin{aligned} d &= \|\text{proj}_{\vec{n_1}} \vec{P_1P_2}\| \\ &= \left\| \frac{-11}{6} (2, -1, -1) \right\| \\ &= \frac{11}{6} \|(2, -1, -1)\| = \frac{11}{6} \sqrt{2^2 + (-1)^2 + (-1)^2} \\ &= \frac{11\sqrt{6}}{6} \end{aligned}$$