Dawson College: **Linear Algebra**: 201-105-05-S3: Fall 2016

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Quiz 12

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Supplementary Exercises #1.4.1 *(6 marks)* Write the parametric equation of the line that passes through the point of intersection and orthogonal of both lines, where $\vec{x} = (2,1,1) + t(5,1,-2)$ and $\vec{x} = (-2,-1,2) + t(3,1,-1)$.

Since the intersection might not occur for the same t value. Let s be the parameter of the second line.

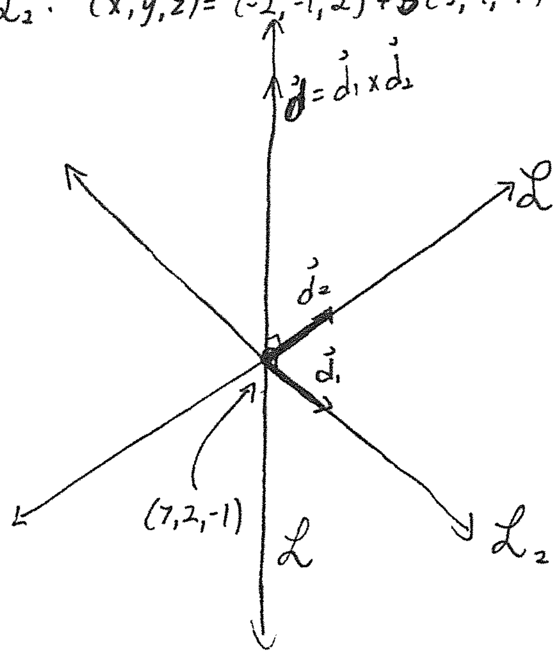
$\mathcal{L}_1: (x,y,z) = (2,1,1) + t(5,1,-2)$
$\mathcal{L}_2: (x,y,z) = (-2,-1,2) + s(3,1,-1)$

Letting the two lines be equal, we get

① $2+5t = -2+3s$
② $1+t = -1+s$
③ $1-2t = 2-s$

Solve for t by ②+③: $2-t = 1$, $-t = -1$, $t = 1$

sub into ②: $1+1 = -1+s$, $3 = s$



Check consistency by subbing into ①

$2+5(1) \stackrel{?}{=} -2+3(3)$
$7 = 7$

$\therefore$ the two lines intersect at $(x,y,z) = (2,1,1) + 1(5,1,-2) = (7,2,-1)$

$\vec{d} = \vec{d}_1 \times \vec{d}_2 = \left(\begin{vmatrix} 1 & 1 \\ -2 & -1 \end{vmatrix}, -\begin{vmatrix} 5 & 3 \\ -2 & -1 \end{vmatrix}, \begin{vmatrix} 5 & 3 \\ 1 & 1 \end{vmatrix} \right) = (1,-1,2)$

$\begin{matrix} 5 & 3 \\ 1 & 1 \\ -2 & -1 \end{matrix}$

$\therefore \mathcal{L}: (x,y,z) = (7,2,-1) + t(1,-1,2).$

Question 2. §3.5 #18 *(3 marks)* Find the volume of the parallelepiped with sides $\vec{u} = (3,1,2)$, $\vec{v} = (4,5,1)$, and $\vec{w} = (1,2,4)$

$$V = |\vec{u} \cdot (\vec{v} \times \vec{w})| = \left| \begin{vmatrix} 3 & 1 & 2 \\ 4 & 5 & 1 \\ 1 & 2 & 4 \end{vmatrix} \right| = \left| 3(-1)^{1+1} \begin{vmatrix} 5 & 1 \\ 2 & 4 \end{vmatrix} + 1(-1)^{1+2} \begin{vmatrix} 4 & 1 \\ 1 & 4 \end{vmatrix} + 2(-1)^{1+3} \begin{vmatrix} 4 & 5 \\ 1 & 2 \end{vmatrix} \right|$$

$$= |3(18) - (15) + 2(3)|$$

$$= 45$$

Question 3. §3.5 #26c *(1 mark)* Suppose that $\vec{u} \cdot (\vec{v} \times \vec{w}) = 3$. Find $\vec{v} \cdot (\vec{w} \times \vec{w}) = \vec{v} \cdot (\vec{0}) = 0$