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Quiz 2

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.2 #3c *(3 marks)* In each part, suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given row echelon form. Solve the system.

$$\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{matrix}$$
$$\begin{bmatrix} 1 & 7 & -2 & 0 & -8 & -3 \\ 0 & 0 & 1 & 1 & 6 & 5 \\ 0 & 0 & 0 & 1 & 3 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Let $x_2 = s$

$x_5 = t$

sub. into

$x_1 + 7x_2 - 2x_3 - 8x_5 = -3$

$x_3 + x_4 + 6x_5 = 5$

$x_4 + 3x_5 = 9$

we obtain

① $x_1 + 7s - 2x_3 - 8t = -3$

② $x_3 + x_4 + 6t = 5$

③ $x_4 + 3t = 9$

From ③ we get $x_4 = 9 - 3t$

and sub into ②

$x_3 + (9 - 3t) + 6t = 5$

$x_3 = -4 - 3t$

sub x_3 and x_4 into ①

$x_1 + 7s - 2(-4 - 3t) - 8t = -3$

$x_1 = -11 + 2t - 7s$

∴ $x_1 = -11 + 2t - 7s$

$x_2 = s$

$x_3 = -4 - 3t$

$x_4 = 9 - 3t$

$x_5 = t$

Question 2. §1.2 #2d *(2 marks)* In each part, suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given reduced row echelon form. Solve the system.

$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 7 \end{bmatrix}$$

∴ $x_1 = -3$

$x_2 = 0$

$x_3 = 7$

Question 3. §1.2 #28 *(5 marks)* Determine the values of a for which the system has no solutions, exactly one solution, or infinitely many solutions

$$\begin{array}{rcrcrcl} x & + & y & + & 7z & = & -7 \\ 2x & + & 3y & + & 17z & = & -16 \\ x & + & 2y & + & (a^2+1)z & = & 3a \end{array}$$

$$\left[\begin{array}{cccc} 1 & 1 & 7 & -7 \\ 2 & 3 & 17 & -16 \\ 1 & 2 & a^2+1 & 3a \end{array}\right]$$

$$\sim \begin{array}{l} -2R_1+R_2\to R_2 \\ -R_1+R_3\to R_3 \end{array} \left[\begin{array}{cccc} 1 & 1 & 7 & -7 \\ 0 & 1 & 3 & -2 \\ 0 & 1 & a^2-6 & 3a+7 \end{array}\right]$$

$$\sim \begin{array}{l} \\ -R_2+R_3\to R_3 \end{array} \left[\begin{array}{cccc} 1 & 1 & 7 & -7 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & a^2-9 & 3a+9 \end{array}\right]$$

no solutions:

Leading 1 in constant column: $a^2-9=0$ and $3a+9\neq 0$

$(a-3)(a+3)=0$

$a=3$ $\quad$ $a=-3$ $\quad$ So $a=3$

exactly one solution:

leading 1 in var. column = # var: $a^2-9\neq 0$

$a\neq 3$ and $a\neq -3$

∞ many solutions:

leading 1 in var column < # var: $a^2-9=0$ and $3a+9=0$

$(a+3)(a-3)=0$ $\quad$ $a=-3$

$a=-3$ $\quad$ $a=3$

So $a=-3$.