Dawson College: Linear Algebra: 201-105-05-S3: Fall 2016

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Quiz 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.3 Consider the matrices

$$A=\begin{bmatrix}3&0\\-1&2\\1&1\end{bmatrix},\ B=\begin{bmatrix}4&-1\\0&2\end{bmatrix},\ C=\begin{bmatrix}1&4&2\\3&1&5\end{bmatrix},\ D=\begin{bmatrix}1&5&2\\-1&0&1\\3&2&4\end{bmatrix},\ E=\begin{bmatrix}6&1&3\\-1&1&2\\4&1&3\end{bmatrix}$$

In each part, compute the given expression (where possible).

#5k. *(3 marks)* $\operatorname{tr}(C^TA^T+2E^T)$

$$= tr\left(\begin{bmatrix}1&3\\4&1\\2&5\end{bmatrix}\begin{bmatrix}3&-1&1\\0&2&1\end{bmatrix}+2\begin{bmatrix}6&-1&4\\1&1&1\\3&2&3\end{bmatrix}\right)$$

$$= tr\left(\begin{bmatrix}3&5&4\\12&-2&5\\6&8&7\end{bmatrix}+2\begin{bmatrix}6&-1&4\\1&1&1\\3&2&3\end{bmatrix}\right)= tr\left(\begin{bmatrix}15&3&12\\14&0&7\\12&12&13\end{bmatrix}\right)=15+0+13$$
$$=28$$

Question 2. §1.3 #21 *(3 marks)* Prove: If A and B are $n\times n$ matrices, then

$\operatorname{tr}(A+B)=\operatorname{tr}(A)+\operatorname{tr}(B)$ Let $A=[a_{ij}]_{n\times n}$, $B=[b_{ij}]_{n\times n}$

$$tr(A+B)=tr([a_{ij}]_{n\times n}+[b_{ij}]_{n\times n})=tr([a_{ij}+b_{ij}]_{n\times n})$$
$$=a_{11}+b_{11}+a_{22}+b_{22}+\cdots+a_{nn}+b_{nn}$$
$$=a_{11}+a_{22}+\cdots+a_{nn}+b_{11}+b_{22}+\cdots+b_{nn}$$
$$=tr(A)+tr(B)$$

Question 3. §1.4 #12 *(4 marks)* Use matrices $A=\begin{bmatrix}3&1\\5&2\end{bmatrix}$ and $B=\begin{bmatrix}2&-3\\4&4\end{bmatrix}$ to verify $(AB)^{-1}=B^{-1}A^{-1}$.

$$AB=\begin{bmatrix}3&1\\5&2\end{bmatrix}\begin{bmatrix}2&-3\\4&4\end{bmatrix}=\begin{bmatrix}10&-5\\18&-7\end{bmatrix}\qquad (AB)^{-1}=\frac{1}{10(-7)+5(18)}\begin{bmatrix}-7&5\\-18&10\end{bmatrix}=\frac{1}{20}\begin{bmatrix}-7&5\\-18&10\end{bmatrix}=LHS$$

$$A^{-1}=\frac{1}{6-5}\begin{bmatrix}2&-1\\-5&3\end{bmatrix}=\begin{bmatrix}2&-1\\-5&3\end{bmatrix}\qquad B^{-1}A^{-1}=\frac{1}{20}\begin{bmatrix}4&3\\-4&2\end{bmatrix}\begin{bmatrix}2&-1\\-5&3\end{bmatrix}=\frac{1}{20}\begin{bmatrix}-7&5\\-18&10\end{bmatrix}=RHS$$

$$B^{-1}=\frac{1}{2(4)+3(4)}\begin{bmatrix}4&3\\-4&2\end{bmatrix}=\frac{1}{20}\begin{bmatrix}4&3\\-4&2\end{bmatrix}\qquad \therefore LHS=RHS.$$