Dawson College: Linear Algebra: 201-105-05-S3: Fall 2016

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Quiz 9

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §2.3 #39 *(3 marks)* Show that if A is a square matrix, then $\det(A^TA) = \det(AA^T)$.

Premise: A is a square matrix

Conclusion: $\det(A^TA) = \det(AA^T)$

LHS $= \det(A^TA)$

$= \det(A^T)\det(A)$ since A is square

$= \det(A)\det(A^T)$ since real numbers commute

$= \det(AA^T)$ since A^T and A are the same dim.

$=$ RHS

Question 2. §2.3 #21 *(4 marks)* Decide whether the given matrix is invertible, and if so, use the adjoint method to find its inverse.

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$$

$\det A = 2(1)(2) = 4$

$C_{11} = (-1)^{1+1}\begin{vmatrix} 1 & -3 \\ 0 & 2 \end{vmatrix} = 2$

$C_{12} = (-1)^{1+2}\begin{vmatrix} 0 & -3 \\ 0 & 2 \end{vmatrix} = 0$

$C_{13} = (-1)^{1+3}\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$

$C_{21} = (-1)^{2+1}\begin{vmatrix} -3 & 5 \\ 0 & 2 \end{vmatrix} = 6$

$C_{22} = (-1)^{2+2}\begin{vmatrix} 2 & 5 \\ 0 & 2 \end{vmatrix} = 4$

$C_{23} = (-1)^{2+3}\begin{vmatrix} 2 & -3 \\ 0 & 0 \end{vmatrix} = 0$

$C_{31} = (-1)^{3+1}\begin{vmatrix} -3 & 5 \\ 1 & -3 \end{vmatrix} = 4$

$C_{32} = (-1)^{3+2}\begin{vmatrix} 2 & 5 \\ 0 & -3 \end{vmatrix} = 6$

$C_{33} = (-1)^{3+3}\begin{vmatrix} 2 & -3 \\ 0 & 1 \end{vmatrix} = 2$

matrix of cof $= \begin{bmatrix} 2 & 0 & 0 \\ 6 & 4 & 0 \\ 4 & 6 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{\det A}\operatorname{adj}(A)$

$= \frac{1}{4}\begin{bmatrix} 2 & 6 & 4 \\ 0 & 4 & 6 \\ 0 & 0 & 2 \end{bmatrix}$

Question 2. §2.3 #21 *(3 marks)* Solve by Cramer's rule, where it applies.

$$\begin{aligned} 7x_1 - 2x_2 &= 3 \\ 3x_1 + x_2 &= 5 \end{aligned}$$

$\begin{bmatrix} 7 & -2 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$A \quad X = b$

$\det A = 13$

$\det(A_1) = \det\left(\begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix}\right) = 13$

$\det(A_2) = \det\left(\begin{bmatrix} 7 & 3 \\ 3 & 5 \end{bmatrix}\right) = 26$

$x_1 = \frac{\det A_1}{\det A} = \frac{13}{13} = 1$

$x_2 = \frac{\det A_2}{\det A} = \frac{26}{13} = 2$