**Dawson College**: **Linear Algebra**: 201-105-04-S03: Fall 2016

**Name:** ____________________

# Test 2

This test is graded out of 40 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** Given

$$A = \begin{bmatrix} 10 & 9 & 1 & 1 & 9 & 3 & 4 & 1 & 0 & 9 \\ 0 & 9 & 1 & 1 & 4 & 9 & 2 & 7 & 7 & 9 \\ 0 & 0 & 8 & 1 & 1 & 4 & 9 & 2 & 7 & 7 \\ 0 & 0 & 0 & 7 & 1 & 1 & 4 & 9 & 2 & 7 \\ 0 & 0 & 0 & 0 & 6 & 1 & 1 & 4 & 9 & 2 \\ 0 & 0 & 0 & 0 & 0 & 5 & 1 & 1 & 4 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 12 & 2 & 2 & 8 & 18 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad C = \begin{bmatrix} b & 2a-3b \\ d & 2c-3d \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & -2 \\ 0 & 2 & 2 \\ 4 & 14 & -8 \end{bmatrix}$$

a. *(2 marks)* Evaluate $\det(A)$.

b. *(2 marks)* True or false: $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, justify?

c. *(2 marks)* Evaluate $\det(D)$.

c. *(2 marks)* Determine the conditions on the $b_i$'s, if any, in order to guarantee that the system

$$D \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

is consistent.

d. *(4 marks)* If $\det(C) = 5$ then determine $\det(B)$.

**Question 2.** Given

$$A = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 3 & 0 & 0 & 4 \\ 5 & 0 & 0 & 6 \\ 0 & 7 & 8 & 0 \end{bmatrix}$$

and a $4 \times 4$ matrix $B$ such that $\det(B) = 101$.

a. *(4 marks)* Evaluate $\det(A)$.

b. *(4 marks)* Evaluate $\det(2A^2B^3)$.

c. *(4 marks)* Evaluate $\det(\operatorname{adj}((3A^{-1})^T))$.

**Question 3.** *(5 marks)* Find the inverse of $A$ by using the adjoint of $A$.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}.$$

**Question 4.** A square matrix $A$ is called *skew-symmetric* if $A^T = -A$.

a. *(3 marks)* Prove: If $A$ is an invertible skew-symmetric matrix, then $A^{-1}$ is skew-symmetric.

c. *(2 marks)* Prove: If $A$ is an $n \times n$ skew-symmetric matrix where $n$ is odd, then $\det(A) = 0$.

**Question 5.** *(6 marks)* Determine for which value(s) of $\lambda$, if any, the following system can be solved using Cramer's Rule.

$$\begin{array}{rcrcrcrcr} x & - & 2y & + & z & - & 2w & = & 5 \\ 3x & - & 4y & & & + & 3w & = & 6 \\ & & 5y & & & - & \lambda w & = & 7 \\ 2x & - & \lambda y & - & 2z & & & = & 8 \end{array}$$

**Bonus Question.** *(3 marks)*
Show that

$$\det(A) = \frac{1}{2}\begin{vmatrix} \operatorname{tr}(A) & 1 \\ \operatorname{tr}(A^2) & \operatorname{tr}(A) \end{vmatrix}$$

for every $2 \times 2$ matrix $A$.