Dawson College: **Linear Algebra**: 201-105-04-S03: Fall 2016

Name: ____________________

Test 3

This test is graded out of 42 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Given $\vec{u} = (-1, \lambda, -3)$, $A(1,0,1)$, $B(0,1,2)$ and $C(3,-2,1)$.

a. *(2 marks)* For which value(s) of λ, if any, $\vec{u}$ is parallel to $\vec{AB}$.

b. *(2 marks)* For which value(s) of λ, if any, $\vec{u}$ is orthogonal to $\vec{AC}$.

c. *(2 marks)* Compute $\text{proj}_{\vec{AB} \times \vec{AC}}(2,3,-4)$.

d. *(1 mark)* Compute the area of the triangle defined by A, B, C.

e. *(1 mark)* Compute the volume of the parallelepiped defined by $\vec{AC}$, $\vec{AB}$ and $\vec{v} = (1,0,0)$.

Question 2. *(5 marks)* Write the parametric equation of the line that passes through the point of intersection and orthogonal to both lines, where

$$\vec{x} = \begin{cases} x = t \\ y = -2 + 2t \\ z = 1 + t \end{cases} \quad t \in \mathbb{R} \qquad \text{and} \qquad \vec{x} = \begin{cases} x = 2 + s \\ y = 2 - s \\ z = 3 + 2s \end{cases} \quad s \in \mathbb{R}.$$

Question 3. *(5 marks)* Find the angle between $\vec{u} = (1, 2, 3)$ and $\vec{v} = (1, 0, 1)$.

Question 4. *(5 marks)* Give the equation of the plane that contains the point $(2,-6,1)$ and the line $(x,y,z) = (2+5t, 2+2t, 1+2t), \quad t \in \mathbb{R}$.

Question 5. *(5 marks)* Using projections find the distance between the given parallel planes.

$$2x - y - z = 5 \text{ and } -4x + 2y + 2z = 12$$

Question 6. *(5 marks)* Maximize $Z = 2x + y + 3z$ subject to the constraints

$$2x - y + z \leq 100$$
$$x + y + 2z \leq 70.$$

Question 7. *(5 marks)* Minimize $Z = x + y$ subject to the constraints

$$\begin{aligned} x + y &\geq 2 \\ 3x + y &\geq 4. \end{aligned}$$

Question 8. If $\vec{u},\ \vec{v}, \vec{w} \in \mathbb{R}^3$ and $\vec{u} \cdot (\vec{v} \times \vec{w}) = 2$ then evaluate

a. *(2 marks)*

$$\vec{u} \cdot (\vec{v} \times \vec{v})$$

b. *(2 marks)*

$$(3\vec{v}) \cdot ((5\vec{u}) \times \vec{w})$$

Bonus Question. *(3 marks)*
Prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and half as long.