Name: Y. Lamontagne

Quiz 10

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

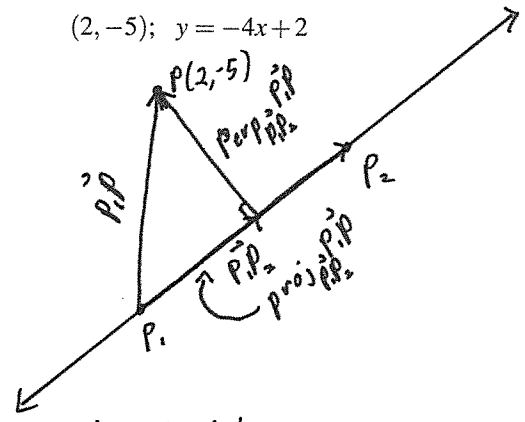
Question 1. §3.4 #6 *(2 marks)* Use the given equation of a line to find a point on the line and a vector parallel to the line.

$(x,y,z) = (4t,7,4+3t)$ = $(0,7,4) + t(4,0,3)$

∴ point on the line: $(0,7,4)$
direction vector: $(4,0,3)$

Question 2. §3.3 #31 *(4 marks)* Using projections find the distance between the point and the line.

$(2,-5)$; $y=-4x+2$



$\vec{P_1P} = P - P_1 = (2,-5) - (0,2) = (2,-7)$

$\vec{P_1P_2} = P_2 - P_1 = (1,-2) - (0,2) = (1,-4)$

$\text{perp}_{\vec{P_1P_2}} \vec{P_1P} = \vec{P_1P} - \text{proj}_{\vec{P_1P_2}} \vec{P_1P} = \vec{P_1P} - \frac{\vec{P_1P}\cdot\vec{P_1P_2}}{\vec{P_1P_2}\cdot\vec{P_1P_2}}\vec{P_1P_2}$

$= (2,-7) - \frac{(2,-7)\cdot(1,-4)}{(1,-4)\cdot(1,-4)}(1,-4)$

$= (2,-7) - \frac{30}{17}(1,-4)$

$= \left(\frac{4}{17}, \frac{1}{17}\right)$

Let's find P_1: let $x=0$
$y = -4(0)+2 = 2$
$\therefore P_1(0,2)$

P_2: let $x=1$
$y = -4(1)+2 = -2$
$\therefore P_2(1,-2)$

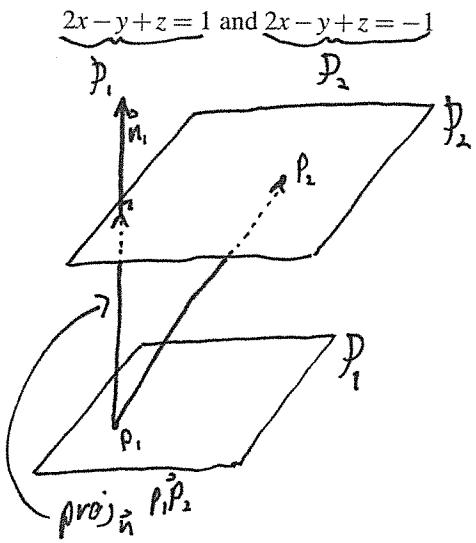
distance $= \| \text{perp}_{\vec{P_1P_2}} \vec{P_1P} \|$

$= \left\| \left(\frac{4}{17}, \frac{1}{17}\right) \right\|$

$= \frac{1}{17}\|(4,1)\| = \frac{\sqrt{17}}{17}$

Question 3. §3.3 #40 *(4 marks)* Find the distance between the given parallel planes.

$2x-y+z=1$ and $2x-y+z=-1$



Lets find P_2: $x=y=0$
$2(0)-0+z=-1$
$z=-1$
$\therefore P_2(0,0,-1)$

$\vec{P_1P_2} = P_2 - P_1$
$= (0,0,-1) - (0,0,1)$
$= (0,0,-2)$

$\text{proj}_{\vec{n}} \vec{P_1P_2} = \frac{\vec{n}\cdot\vec{P_1P_2}}{\vec{n}\cdot\vec{n}}\vec{n}$

$= \frac{(2,-1,1)\cdot(0,0,-2)}{(2,-1,1)\cdot(2,-1,1)}(2,-1,1)$

$= \frac{-2}{6}(2,-1,1) = -\frac{1}{3}(2,-1,1)$

Lets find P_1: let $x=y=0$
$2(0)-(0)+z=1$
$z=1$
$\therefore P_1(0,0,1)$

distance $= \| \text{proj}_{\vec{n}} \vec{P_1P_2} \|$

$= \left\| -\frac{1}{3}(2,-1,1) \right\| = \frac{1}{3}\sqrt{2^2+(-1)^2+1^2} = \frac{\sqrt{6}}{3}$