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Quiz 11

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §3.5 #15 *(3 marks)* Find the area of a triangle in 3-space that has the given vertices

$P_1(2,6,-1), P_2(1,1,1), P_3(4,6,2)$
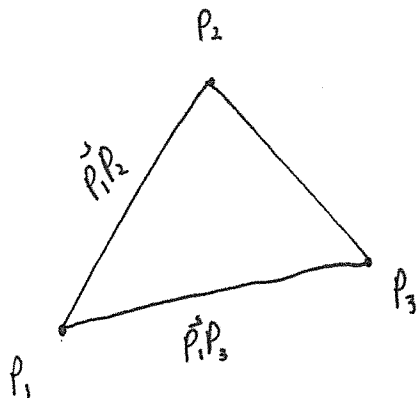
$$\vec{P_1P_2} = P_2 - P_1$$
$$= (1,1,1) - (2,6,-1) = (-1,-5,2)$$
$$\vec{P_1P_3} = P_3 - P_1$$
$$= (4,6,2) - (2,6,-1) = (2,0,3)$$

$$\vec{P_1P_2} \times \vec{P_1P_3} = \left(\begin{vmatrix} -5 & 0 \\ 2 & 3 \end{vmatrix}, -\begin{vmatrix} -1 & 2 \\ 2 & 3 \end{vmatrix}, \begin{vmatrix} -1 & 2 \\ -5 & 0 \end{vmatrix} \right) = (-15, 7, 10)$$

$$\begin{matrix} -1 & 2 \\ -5 & 0 \\ 2 & 3 \end{matrix}$$

$$\text{Area} = \frac{\|\vec{P_1P_2} \times \vec{P_1P_3}\|}{2} = \frac{\|(-15, 7, 10)\|}{2}$$

$$= \frac{\sqrt{(-15)^2 + 7^2 + 10^2}}{2} = \frac{\sqrt{374}}{2}$$

Question 2. §3.5 #17 *(3 marks)* Find the volume of the parallelepiped with sides $\vec{u} = (2,-6,2), \vec{v} = (0,4,-2)$, and $\vec{w} = (2,2,-4)$

$$\text{volume} = |\vec{u} \cdot (\vec{v} \times \vec{w})|$$
$$= \left| \begin{vmatrix} 2 & -6 & 2 \\ 0 & 4 & -2 \\ 2 & 2 & -4 \end{vmatrix} \right|$$
$$= |0C_{21} + 4C_{22} - 2C_{23}|$$
$$= \left| 4(-1)^{2+2} \begin{vmatrix} 2 & 2 \\ 2 & -4 \end{vmatrix} - 2(-1)^{2+3} \begin{vmatrix} 2 & -6 \\ 2 & 2 \end{vmatrix} \right|$$

$$= |4(-8-4) + 2(4+12)|$$
$$= |-4(12) + 2(16)|$$
$$= |-48 + 32|$$
$$= 16$$

Question 3. §3.5 Suppose that $\vec{u} \cdot (\vec{v} \times \vec{w}) = 3$. Find

25b *(2 marks)* $(\vec{v} \times \vec{w}) \cdot \vec{u} = \vec{u} \cdot (\vec{v} \times \vec{w})$ since the dot product is commutative

$$= 3$$

26a *(2 marks)* $\vec{v} \cdot (\vec{u} \times \vec{w}) = \begin{vmatrix} v_1 & v_2 & v_3 \\ u_1 & u_2 & u_3 \\ w_1 & w_2 & w_3 \end{vmatrix} \underset{R_1 \leftrightarrow R_2}{=} -\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = -3$