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Quiz 2

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.2 #3b *(3 marks)* In each part, suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given row echelon form. Solve the system.

$$\begin{bmatrix} 1 & 0 & 8 & -5 & 6 \\ 0 & 1 & 4 & -9 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$$

Let $x_4 = t \quad t \in \mathbb{R}$

sub into

$$\begin{aligned} x_1 \quad + 8x_3 - 5x_4 &= 6 \\ x_2 + 4x_3 - 9x_4 &= 3 \\ x_3 + x_4 &= 2 \end{aligned}$$

we obtain

$$\begin{aligned} ① \quad x_1 \quad + 8x_3 - 5t &= 6 \\ ② \quad x_2 + 4x_3 - 9t &= 3 \\ ③ \quad x_3 + t &= 2 \end{aligned}$$

From ③ we get $x_3 = 2 - t$

sub into ②

$$x_2 + 4(2-t) - 9t = 3$$
$$x_2 = -5 + 13t$$

sub into ①

$$x_1 \quad + 8(2-t) - 5t = 6$$
$$x_1 = -16 + 13t + 6$$
$$x_1 = -10 + 13t$$

$\therefore \quad x_1 = -10 + 13t$

$x_2 = -5 + 13t$

$x_3 = 2 - t$

$x_4 = t$

Question 2. §1.2 #3d *(2 marks)* In each part, suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given row echelon form. Solve the system.

$$\begin{bmatrix} 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From the last row we get $0x_1 + 0x_2 + 0x_3 = 1$

$0 = 1$

$\therefore$ no value can satisfy the above

$\therefore$ no solution

$\therefore$ the system is inconsistent.

Question 3. §1.2 #26 *(5 marks)* Determine the values of a for which the system has no solutions, exactly one solution, or infinitely many solutions

$$\begin{array}{rcrcrcl} x & + & 2y & + & z & = & 2 \\ 2x & - & 2y & + & 3z & = & 1 \\ x & + & 2y & - & (a^2-3)z & = & a \end{array}$$

$$\left[\begin{array}{cccc} 1 & 2 & 1 & 2 \\ 2 & -2 & 3 & 1 \\ 1 & 2 & -(a^2-3) & a \end{array}\right]$$

$$\sim \begin{array}{r} \\ -2R_1+R_2 \to R_2 \\ -R_1+R_3 \to R_3 \end{array} \left[\begin{array}{cccc} 1 & 2 & 1 & 2 \\ 0 & -6 & 1 & -3 \\ 0 & 0 & -a^2+2 & a-2 \end{array}\right]$$

$$= \left[\begin{array}{cccc} 1 & 2 & 1 & 2 \\ 0 & -6 & 1 & -3 \\ 0 & 0 & 2-a^2 & a-2 \end{array}\right]$$

<u>no solutions:</u> Leading 1 in constant column:

if $2-a^2=0$ and $a-2 \neq 0$

$2 = a^2 \qquad a \neq 2$

$\pm\sqrt{2} = a$

if $a = \pm\sqrt{2}$ then no solutions

<u>unique solution:</u> # leading 1 (in var. column) = # var

$2-a^2 \neq 0$

$2 \neq a^2$

$\pm\sqrt{2} \neq a$

$\therefore$ unique solution if $a \neq \pm\sqrt{2}$

<u>infinitely many solutions:</u> # leading 1 < # var

if $2-a^2=0$ and $a-2=0$

$a = \pm\sqrt{2} \qquad a = 2$

impossible

$\therefore$ not possible to get ∞ many solutions.