

Quiz 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.4 #16 (4 marks) Use the given information to find A.

$$(5A^T)^{-1} = \begin{bmatrix} -3 & -1 \\ 5 & 2 \end{bmatrix}$$

$$\left[(5A^T)^{-1} \right]^{-1} = \begin{bmatrix} -3 & -1 \\ 5 & 2 \end{bmatrix}^{-1}$$

$$5A^T = \frac{1}{(-3)(2) - (-1)(5)} \begin{bmatrix} 2 & 1 \\ -5 & -3 \end{bmatrix}$$

$$5A^T = \frac{1}{-1} \begin{bmatrix} 2 & 1 \\ -5 & -3 \end{bmatrix}$$

$$5A^T = \begin{bmatrix} -2 & -1 \\ 5 & 3 \end{bmatrix}$$

$$5A^T = \begin{bmatrix} -2 & -1 \\ 5 & 3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} -2/5 & -1/5 \\ 1 & 3/5 \end{bmatrix}$$

$$(A^T)^T = \begin{bmatrix} -2/5 & -1/5 \\ 1 & 3/5 \end{bmatrix}^T$$

$$A = \begin{bmatrix} -2/5 & 1 \\ -1/5 & 3/5 \end{bmatrix}$$

Question 2. §1.4 #30 (3 marks) Assuming that all matrices are $n \times n$ are invertible, solve for D.

$$ABC^T DBA^T C = AB^T$$

$$(ABC^T)^{-1} (ABC^T) D BA^T C (BA^T C)^{-1} = (ABC^T)^{-1} AB^T (BA^T C)^{-1}$$

$$I D I$$

$$D = (C^T)^{-1} B^{-1} A^{-1} A B^T C^{-1} (A^T)^{-1} B^{-1}$$

$$D = (C^T)^{-1} B^{-1} B^T C^{-1} (A^T)^{-1} B^{-1}$$

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Question 2. §1.4 #54b (3 marks) A square matrix A is said to be *idempotent* if $A^2 = A$. Show that if A is idempotent, then $2A - I$ is invertible and is its own inverse.

Premise:

- A is idempotent, $A^2 = A$

Conclusion:

- $(2A - I)$ is invertible and is its own inverse.

Need to show:

$$(2A - I)(2A - I) = I$$

$$\text{LHS} = (2A - I)(2A - I)$$

$$= 4A^2 - 2A - 2A + I$$

$$= 4A - 2A - 2A + I \text{ by premise}$$

$$= I = \text{RHS}$$