Dawson College: **Linear Algebra**: 201-105-05-S3: Fall 2016

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Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.4 #54a *(3 marks)* A square matrix A is said to be *idempotent* if $A^2 = A$. Show that if A is idempotent, then so is $I - A$.

premise:

- A is idempotent $\Rightarrow A^2 = A$

Conclusion:

- $I - A$ is idempotent

Need to show $(I-A)^2 = (I-A)$

$$\begin{aligned} LHS &= (I-A)^2 \\ &= (I-A)(I-A) \\ &= I - I\cdot A - A\cdot I + A\cdot A \\ &= I - A - A + A^2 \\ &= I - A - A + A \quad \text{by premise} \\ &= I - A \\ &= RHS \end{aligned}$$

Question 2. §1.4 #33 *(2 marks)* Simplify:

$$(AC^{-1})^{-1}(AC^{-1})(AC^{-1})^{-1}AD^{-1}$$

$$= (C^{-1})^{-1}A^{-1}AC^{-1}(C^{-1})^{-1}A^{-1}AD^{-1}$$

$$= CIC^{-1}CID^{-1}$$

$$= CID^{-1} = CD^{-1}$$

Question 3. §1.5 #32 *(5 marks)* Write the given matrix as a product of elementary matrices.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \sim_{-R_1+R_2\to R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \sim_{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \sim_{-R_3+R_2\to R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim_{-R_2+R_1\to R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

From the above 4 elem. row. op. we can obtain 4 elem. matrices.

s.t. $E_4E_3E_2E_1A = I$

$$(E_4E_3E_2E_1)^{-1}E_4E_3E_2E_1A = (E_4E_3E_2E_1)^{-1}$$

$$IA = E_1^{-1}E_2^{-1}E_3^{-1}E_4^{-1}$$

$$A = E_1^{-1}E_2^{-1}E_3^{-1}E_4^{-1}$$

where $E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, $E_4^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$