**Dawson College**: **Linear Algebra**: 201-105-05-S3: Fall 2016

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# Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** §1.6 #16 *(5 marks)* Determine conditions on the $b_i$'s, if any, in order to guarantee that the linear system is consistent.

$$\begin{array}{rcrcrcl} x_1 & - & 2x_2 & - & x_3 & = & b_1 \\ -4x_1 & + & 5x_2 & + & 2x_3 & = & b_2 \\ -4x_1 & + & 7x_2 & + & 4x_3 & = & b_3 \end{array}$$

$$\begin{bmatrix} 1 & -2 & -1 & b_1 \\ -4 & 5 & 2 & b_2 \\ -4 & 7 & 4 & b_3 \end{bmatrix}$$

$$\sim \begin{array}{l} \\ 4R_1+R_2 \to R_2 \\ 4R_1+R_3 \to R_3 \end{array} \begin{bmatrix} 1 & -2 & -1 & b_1 \\ 0 & -3 & -2 & b_2+4b_1 \\ 0 & -1 & 0 & b_3+4b_1 \end{bmatrix}$$

$$\sim R_2 \leftrightarrow R_3 \begin{bmatrix} 1 & -2 & -1 & b_1 \\ 0 & -1 & 0 & b_3+4b_1 \\ 0 & -3 & -2 & b_2+4b_1 \end{bmatrix}$$

$$\sim -3R_2+R_3 \to R_3 \begin{bmatrix} 1 & -2 & -1 & b_1 \\ 0 & -1 & 0 & b_3+4b_1 \\ 0 & 0 & -2 & b_2-3b_3-8b_1 \end{bmatrix}$$

The RREF of is I. ∴ by TFAE, the system is consistent for all $b_i$

**Question 2.** §1.7 #23 *(2 marks)* Find all values of the unknown constant(s) in order for $A$ to be symmetric.

$$\begin{bmatrix} 4 & -3 \\ a+5 & -1 \end{bmatrix}$$

For the matrix to be symmetric $A^T = A$.

$$\begin{bmatrix} 4 & a+5 \\ -3 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ a+5 & -1 \end{bmatrix} \quad \therefore \begin{array}{l} -3 = a+5 \\ a = -8 \end{array}$$

**Question 3.** §1.7 #37a *(3 marks)* A square matrix $A$ is called *skew-symmetric* if $A^T = -A$. Prove: If $A$ is an invertible skew-symmetric matrix, then $A^{-1}$ is skew-symmetric.

Premise:
- $A$ is invertible
- $A$ is skew symmetric: $A^T = -A$

Conclusion:
- $A^{-1}$ is skew symmetric

So we need to show $(A^{-1})^T = -A^{-1}$.

$$\begin{aligned} LHS &= (A^{-1})^T \\ &= (A^T)^{-1} \\ &= (-A)^{-1} \quad \text{since } A \text{ is skew symmetric} \\ &= -A^{-1} \\ &= RHS. \end{aligned}$$