Name: ____________________

Test 1

This test is graded out of 40 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1.

a. *(6 marks)* Solve the following system by Gauss-Jordan elimination:

$$\begin{array}{rcrcrcrcrcr} & & y & - & z & + & w & - & v & = & 3 \\ 2x & - & 3y & + & 4z & - & 4w & + & v & = & 2 \\ 3x & - & 3y & + & 4z & - & 4w & + & v & = & 2 \end{array}$$

b. *(1 mark)* Find two particular solutions to the above system.

Question 2. Consider the matrices:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ 1 & -1 \end{bmatrix},\ B = \begin{bmatrix} 2 & -5 & 2 \\ -3 & 2 & 1 \end{bmatrix},\ C = \begin{bmatrix} 1 & 2 \\ 5 & 0 \end{bmatrix},\ D = \begin{bmatrix} -1 & 1 \\ 4 & -3 \end{bmatrix},\ E = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

a. *(2 marks)* Evaluate if possible, justify.

$$\text{trace}(C)A - \text{trace}(D)B^T$$

b. *(2 marks)* Evaluate if possible, justify.

$$EA$$

c. *(2 marks)* Evaluate if possible, justify.

$$(AC)E$$

e. *(4 marks)* Solve for X if possible.

$$CXD = 10I_2$$

.

Question 3. *(5 marks)* Solve for X given that it satisfies

$$\left(2A+X^T\right)^{-1}=I$$

where $A=\begin{bmatrix}1 & 3\\1 & 2\end{bmatrix}$.

Question 4. *(2 marks)* Let A and B be matrices such that AB is defined. Show that if the first and second columns of B are equal then so are the first and second columns of AB.

Question 5. *(3 marks)* Given an $n \times n$ matrix A such that $p(A) = 0$ where $p(x) = x^3 - x^2 + 1$. Determine the inverse of A in terms of A.

Question 6. Consider the following augmented matrix in which $*$ denotes an arbitrary number and $\blacksquare$ denotes a nonzero number. Determine whether the system of the given augmented matrix is consistent. If consistent, is the solution unique?

a. *(2 marks)* $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & \blacksquare & * \end{bmatrix}$

b. *(2 marks)* $\begin{bmatrix} \blacksquare & * & * & * & * & * \\ 0 & \blacksquare & * & * & 0 & * \\ 0 & 0 & 0 & 0 & \blacksquare & 0 \\ 0 & 0 & 0 & 0 & * & \blacksquare \end{bmatrix}$

Question 7. *(3 marks)* Given that A, H and M are invertible matrices simplify the following expression.

$$(AM^{-1})^{-1}(A^2)^T(H^T A^T (A^{-1})^T)^T$$

Question 8. Consider the following system:

$$\begin{array}{rcrcr} x & - & 2y & = & 5 \\ 3x & - & 4y & = & 6 \end{array}$$

a. *(1 mark)* Write the above system as a matrix equation.

b. *(2 marks)* Solve the matrix equation by using the inverse of the coefficient matrix.

c. *(2 marks)* Give a geometrical interpretation of the solution set.

Question 9. *(1 mark)* True or False: a system with more unknowns than equations has at least one solution.

Bonus Question. *(3 marks)*
Prove: If c is a scalar and A is an $n \times n$ matrix then $\mathrm{tr}(cA^T) = c\,\mathrm{tr}(A)$.