Name: ____________________

Test 2

This test is graded out of 40 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Given

$$A = \begin{bmatrix} 10 & 9 & 1 & 1 & 9 & 3 & 4 & 1 & 0 & 9 \\ 0 & 9 & 1 & 1 & 4 & 9 & 2 & 7 & 7 & 9 \\ 0 & 0 & 8 & 1 & 1 & 4 & 9 & 2 & 7 & 7 \\ 0 & 0 & 0 & 7 & 1 & 1 & 4 & 9 & 2 & 7 \\ 0 & 0 & 0 & 0 & 6 & 1 & 1 & 4 & 9 & 2 \\ 0 & 0 & 0 & 0 & 0 & 5 & 1 & 1 & 4 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \; B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \; C = \begin{bmatrix} c & d \\ -7a-5c & -7b-5d \end{bmatrix}, \; D = \begin{bmatrix} 1 & -1 & -2 \\ -3 & 2 & 6 \\ 4 & 14 & -8 \end{bmatrix}$$

a. *(2 marks)* Evaluate $\det(A)$.

b. *(2 marks)* Is A invertible, justify?

c. *(2 marks)* Evaluate $\det(D)$.

c. *(2 marks)* Is D expressible as a product of elementary matrices, justify?

d. *(4 marks)* If $\det(C) = 2$ then determine $\det(B)$.

Question 2. Given

$$A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 3 & 4 & 0 \\ 0 & 5 & 6 & 0 \\ 7 & 0 & 0 & 8 \end{bmatrix}$$

a. *(4 marks)* Evaluate $\det(A)$.

b. *(4 marks)* Evaluate $\det(\operatorname{adj}((3A^T)^{-1}))$.

Question 3. *(5 marks)* Express the following matrix as a product of elementary matrices

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}.$$

Question 4. Prove: If $A^T A = A$ then

a. *(2 marks)* A is symmetric,

b. *(2 marks)* $A = A^2$,

c. *(2 marks)* $\det(A) = 0$ or $\det(A) = 1$.

Question 6. Consider the following system:

$$\begin{array}{rcrcrcr} x & - & 2y & + & 3z & = & 5 \\ & - & 4y & - & 2z & = & 6 \\ & & & & 3z & = & 1 \end{array}$$

a. *(1 mark)* Write the above system as a matrix equation.

b. *(4 marks)* Find the inverse of the coefficient matrix using the adjoint.

c. *(1 mark)* Solve the system using the inverse of the coefficient matrix.

Question 8. *(3 marks)* Solve the following system using Cramer's Rule, if possible:

$$\begin{array}{rcrcr} x & - & 2y & = & 5 \\ 3x & - & 4y & = & 6 \end{array}$$

Bonus Question. *(3 marks)*
Show that

$$\det(A) = \frac{1}{2}\begin{vmatrix} \mathrm{tr}(A) & 1 \\ \mathrm{tr}(A^2) & \mathrm{tr}(A) \end{vmatrix}$$

for every 2×2 matrix A.