Name: ____________________

Test 3

This test is graded out of 42 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Given $\vec{u} = (-3, \lambda, -1)$, $A(2,0,1)$, $B(1,0,2)$ and $C(3,-2,-1)$.

a. *(2 marks)* For which value(s) of λ, if any, $\vec{u}$ is parallel to $\vec{AB}$.

b. *(2 marks)* For which value(s) of λ, if any, $\vec{u}$ is orthogonal to $\vec{AC}$.

c. *(2 marks)* Compute $\text{proj}_{(2,3,-4)} \vec{AB} \times \vec{AC}$.

d. *(1 mark)* Compute the area of the parallelogram defined by $\vec{AB}$ and $\vec{AC}$.

e. *(1 mark)* Find the equation of the plane containing the points A, B and C.

Question 2. *(5 marks)* Write the parametric equation of the line that passes through the point of intersection and orthogonal of both lines, where $\vec{x} = (2,1,1) + t(5,1,-2),\ t \in \mathbb{R}$ and $\vec{x} = (-2,-1,2) + s(3,1,-1),\ s \in \mathbb{R}$.

Question 3. *(5 marks)* Find the angle between $\vec{u} = (3,2,1)$ and $\vec{v} = (0,2,0)$

Question 4. *(5 marks)* Give the equation of the plane that contains the point $(2,-6,1)$ and orthogonal to both planes: $x-y+z=1$ and $2x+y-z=2$.

Question 5. *(5 marks)* Using projections find the distance between the point and the line.

$$(2,-5); \quad y=-4x+2$$

Question 6. *(5 marks)* Maximize $Z = 3x + y$ subject to the constraints

$$\begin{aligned} 2x - y &\leq 60 \\ x + y &\leq 50. \end{aligned}$$

Question 7. *(5 marks)* Minimize $Z = 2x + y + z$ subject to the constraints

$$\begin{aligned} x + z &\geq 2 \\ 2x + y + z &\geq 3. \end{aligned}$$

Question 8. If $\vec{u},\ \vec{v}, \vec{w} \in \mathbb{R}^3$ and $\vec{u} \cdot (\vec{v} \times \vec{w}) = 3$ then evaluate

a. *(2 marks)*

$$(\vec{u} - \vec{v}) \cdot (\vec{v} \times \vec{w})$$

b. *(2 marks)*

$$(\vec{w} \times \vec{w}) \cdot \vec{v}$$

Bonus Question. *(3 marks)*
Prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and half as long.