Dawson College: **Linear Algebra**: 201-105-05: Fall 2016

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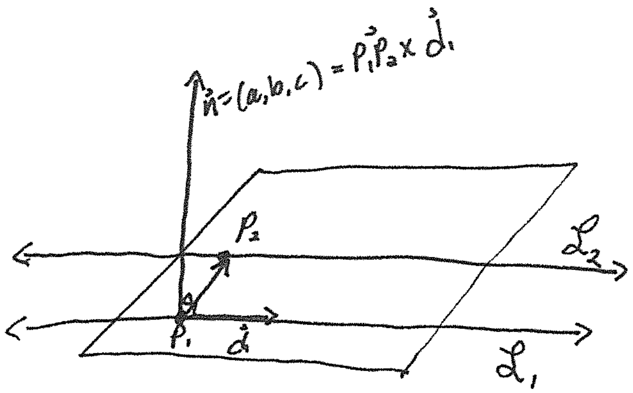
For each part, show all your work and include a sketch.

Question 1. Given the two lines: $\mathscr{L}_1 : (x,y,z) = (1,0,-2)+t(1,3,2), \quad t \in \mathbb{R}$ and $\mathscr{L}_2 : (x,y,z) = (1,2,-2)+t(-2,-6,-4), \quad t \in \mathbb{R}$.

a. Find the parametric and general equation of the plane that contains $\mathscr{L}_1$ and $\mathscr{L}_2$.



$\vec{P_1P_2} = P_2 - P_1 = (1,2,-2)-(1,0,-2)$
$= (0,2,0)$

$\vec{n} = \vec{P_1P_2}\times\vec{d_1} = \left(\begin{vmatrix} 2 & 3 \\ 0 & 2\end{vmatrix}, -\begin{vmatrix} 0 & 1 \\ 0 & 2\end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 2 & 3\end{vmatrix}\right)$

$\begin{matrix} 0 & 1 \\ 2 & 3 \\ 0 & 2 \end{matrix} \quad = (4,0,-2)$

$ax+by+cz=d \Rightarrow 4x-2z=d$

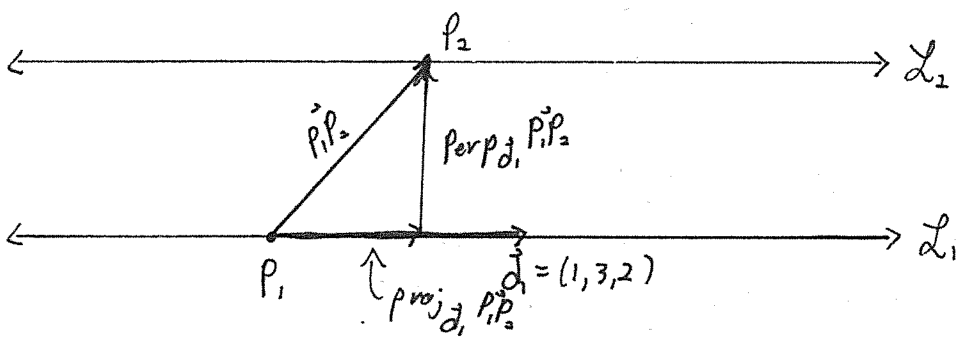
sub in point to solve for d

$4(1)-2(-2)=d$
$8=d$

$\therefore \quad 4x-2z=8$
$2x-z=4$

$(x,y,z) = P_0 + s\vec{d_1} + t\vec{d_2} = (1,0,-2)+s(0,2,0)+t(1,3,2) \quad s,t\in\mathbb{R}$

b. Using projections find the distance from $\mathscr{L}_1$ to $\mathscr{L}_2$.



$perp_{\vec{d_1}}\vec{P_1P_2} = \vec{P_1P_2} - proj_{\vec{d_1}}\vec{P_1P_2}$

$= (0,2,0) - \dfrac{(0,2,0)\cdot(1,3,2)}{(1,3,2)\cdot(1,3,2)}(1,3,2)$

$= (0,2,0) - \dfrac{6}{14}(1,3,2)$

$= (0,2,0) - \dfrac{3}{7}(1,3,2)$

$= \left(-\dfrac{3}{7}, \dfrac{5}{7}, -\dfrac{6}{7}\right) = \dfrac{1}{7}(-3,5,-6)$

$\therefore$ distance $= \|perp_{\vec{d_1}}\vec{P_1P_2}\|$

$= \|\frac{1}{7}(-3,5,-6)\|$

$= \frac{1}{7}\sqrt{(-3)^2+5^2+(-6)^2}$

$= \frac{1}{7}\sqrt{9+25+36}$

$= \dfrac{\sqrt{70}}{7}$

c. Find the equation of the line which passes through $P(1,1,1)$ and is orthogonal to both $\mathscr{L}_1$ and $\mathscr{L}_2$.

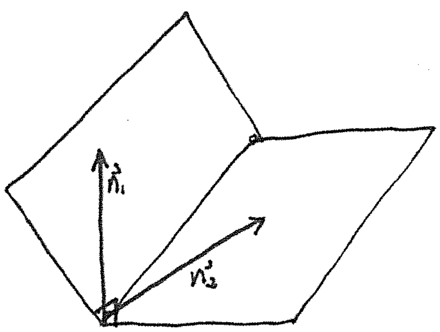
the direction of

$\vec{n}$ from part a. is orthogonal to $\vec{d}_1$ and $\vec{d}_2$.

Hence

$$(x,y,z) = (1,1,1) + t(4,0,-2) \qquad t \in \mathbb{R}.$$

d. Find the angle between the plane found in part a. and the plane $\mathscr{P}: (x,y,z) = (1,2,0) + s(2,-1,1) + t(5,-1,6), \quad s,t \in \mathbb{R}$.



Lets find the normal of the above plane

$$\vec{n}_2 = \vec{d}_1 \times \vec{d}_2 = \left(\begin{vmatrix} -1 & -1 \\ 1 & 6 \end{vmatrix}, -\begin{vmatrix} 2 & 5 \\ 1 & 6 \end{vmatrix}, \begin{vmatrix} 2 & 5 \\ -1 & -1 \end{vmatrix} \right)$$

$$\begin{matrix} 2 & 5 \\ -1 & -1 \\ 1 & 6 \end{matrix} = (-5,-7,3)$$

$$\vec{n}_1 \cdot \vec{n}_2 = \|\vec{n}_1\| \|\vec{n}_2\| \cos\theta$$

$$(4,0,-2)\cdot(-5,-7,3) = \sqrt{4^2+0^2+(-2)^2}\sqrt{(-5)^2+(-7)^2+3^2}\cos\theta$$

$$-20-6 = \sqrt{20}\sqrt{25+49+9}\cos\theta$$

$$-26 = \sqrt{20}\sqrt{83}\cos\theta$$

$$\cos\theta = \frac{-26}{\sqrt{20}\sqrt{83}}$$

$$\theta = \arccos\left(\frac{-26}{\sqrt{20}\sqrt{83}}\right) = 129^\circ \text{ or } 51^\circ$$