Dawson College: **Probability and Statistics**: 201-BZS-05-S01: Fall 2016

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Test 1

This test is graded out of XX marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Below are the final grades of a Calculus II class:

82	34	64	69	92	67	10
38	71	11	70	74	53	99
83	40	56	79	75	67	66
71	53	14	75	75	60	61
80	70	59	46	70	85	70

a. *(1 mark)* Describe the type of data and the level of measurement. (i.e. numerical data, categorical data, continuous data, discrete data, nominal data, etc).

The data is numerical discrete and the level of measurement is ratio.

b. *(1 mark.)* If the above data is from a class that was offered at Dawson College, would it be a random sample of the student population taking Calulus II during that semester? Justify.

Not a random sample since the grades are from a specific class, could be called a convenience sample.

c. *(1 mark.)* If the above data was used for a study would it be an observational study or experimental study? Justify.

The study would be observational as no treatment was applied on the group.

d. *(1 mark)* Would it be appropriate to draw any causal conclusions based on the data? Justify.

It is not appropriate to draw any causal conclusion as there might be several confounding variables to any conclusion.

e. *(2 marks)* Compute the sample mean and standard deviation *(use your calculator to save time.)*.

$\bar{x} = 62.54$

$s = 21.25$

f. *(1 mark)* Suppose there is a mistake in the above data, that the student with a grade of 10 actually had a grade of 99. Recompute the sample mean and standard deviation *(use your calculator to save time.)* with the correction.

$\bar{x} = 65.09$ $\quad s = 20.07$

Question 1. ***continued*** Below are the final grades of a Calculus II class:*(same data)*

82	34	64	69	92	67	10
38	71	11	70	74	53	99
83	40	56	79	75	67	66
71	53	14	75	75	60	61
80	70	59	46	70	85	70

g. *(2 marks)* Sort the data *(you can use a stem and leaf display)*.

stem	leaves
1	0 1 4
2	
3	4 8
4	0 6
5	3 6 3 9
6	4 9 7 7 6 0 1
7	1 0 4 9 5 1 5 5 0 0 0
8	2 3 0 5
9	2 9

10 11 14 34 38 40 46 53 53 56 59 60 61
64 66 67 67 69 70 70 70 70 71 71 74 75
75 75 79 80 82 83 85 92 99

h. *(3 marks)* Determine the 25^{th}, 50^{th}, and 75^{th} percentile.

P_{25} : $\frac{25}{100}(35) = 8.75$ $\therefore$ depth is 9 and $P_{25} = 53$

P_{50} : $\frac{50}{100}(35) = 17.5$ $\therefore$ depth is 18 and $P_{50} = 69$

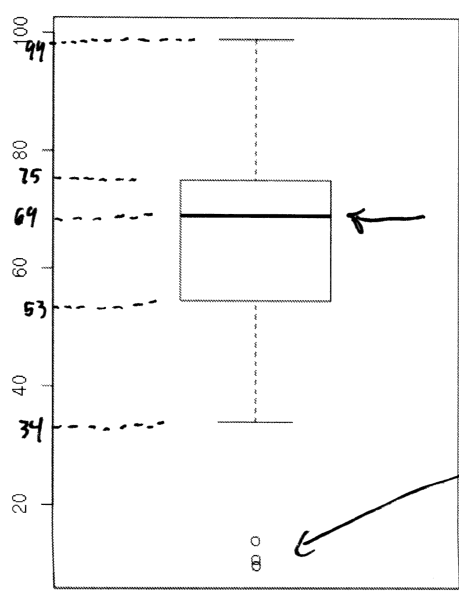
P_{75} : $\frac{75}{100}(35) = 26.25$ $\therefore$ depth is 27 and $P_{75} = 75$

i. *(4 marks)* Sketch a box plot representing the above data. Which grades are possible outliers?

$IQR = P_{75} - P_{25} = 75 - 53 = 22$

max upper whisker: $P_{75} + 1.5\, IQR = 108$

min upper whisker: $P_{25} - 1.5\, IQR = 20$



possible outliers grades of 10, 11, 14

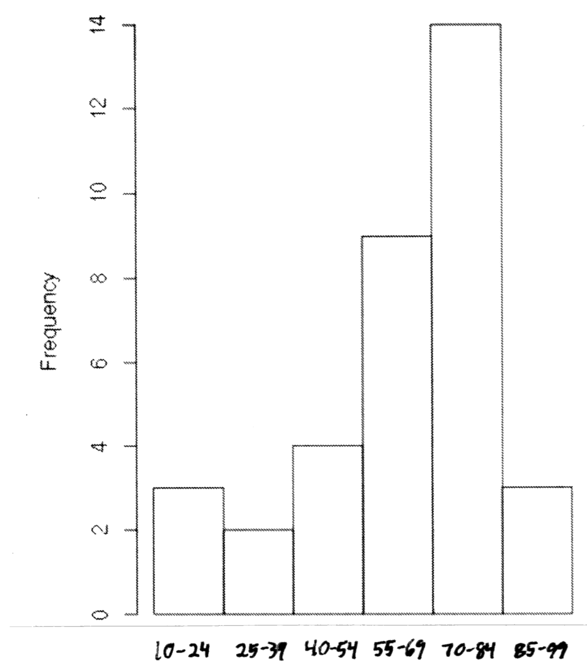
Question 1. ***continued*** Below are the final grades of a Calculus II class: *(same data)*

82	34	64	69	92	67	10
38	71	11	70	74	53	99
83	40	56	79	75	67	66
71	53	14	75	75	60	61
80	70	59	46	70	85	70

j. *(4 marks)* Sketch a histogram representing the above data (use $\sqrt{n}$ to determine the number of classes). Describe the histogram?

class $= \lceil\sqrt{35}\rceil = \lceil 5.9 \rceil = 6$, range $= 99 - 10 = 89$, class width $= \lceil 89/6 \rceil = 15$

class limits	Marks	frequency
10 - 24	17	3
25 - 39	32	2
40 - 54	47	4
55 - 69	57	9
70 - 84	77	14
85 - 99	92	3

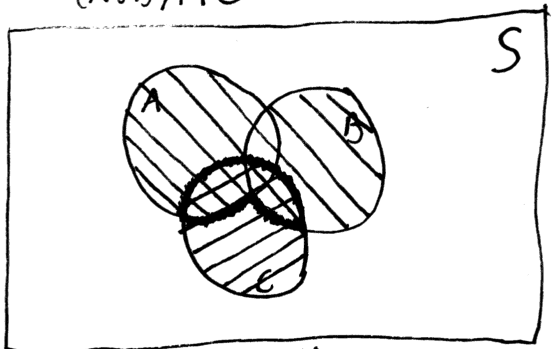


The histogram is skewed left.

Question 2. Given A, B and C are events belonging to a sample space S.

a. *(3 marks)* Sketch the Venn diagrams of $(A \cup B) \cap C$ and $(A \cap C) \cup (B \cap C)$. What conclusion can you draw from the two Venn diagrams?

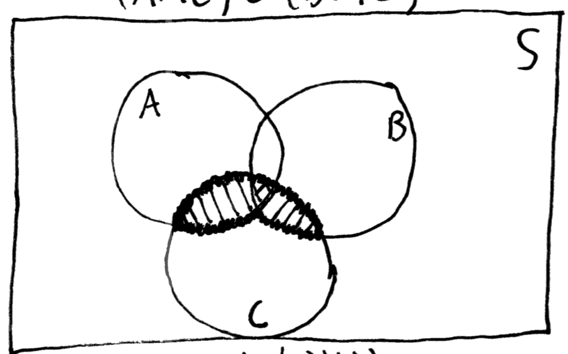
$(A \cup B) \cap C$



$A \cup B$ shaded \\\\

C shaded ////

$(A \cap C) \cup (B \cap C)$



$A \cap C$ shaded \\\\\

$B \cap C$ shaded /////

Both regions are the same. $\therefore (A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

b. *(2 marks)* Prove: $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$. *Hint: You may use the theorem that* $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$ *and part a.*

$$P(A \cup B \cup C) = P((A \cup B) \cup C) = P(A \cup B) + P(C) - P((A \cup B) \cap C)$$

$$= P(A) + P(B) - P(A \cap B) + P(C) - P((A \cup B) \cap C)$$

$$= P(A) + P(B) + P(C) - P(A \cap B) - P((A \cap C) \cup (B \cap C))$$

$$= P(A) + P(B) + P(C) - P(A \cap B) - [P(A \cap C) + P(B \cap C) - P((A \cap C) \cap (B \cap C))$$

$$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Question 3. On a legacy system a password can be generated by using 26 lower case letters, 26 upper case letters, 10 digits $\{0,1,2,3,4,5,6,7,8,9\}$ and the following 22 special character $\{!, ", \#, \$, \%, \&, ', *, +, -, ., /, :, ;, <, =, >, ?, @, [,], |\}$. The password must be of length exactly eight.

a. *(1 mark)* How many different passwords are possible?

$$(26+26+10+22)^8$$

Each of the eight character has 26+26+10+22 possibility

b. *(2 marks)* How many different passwords are possible with the additional condition that the password cannot begin or end with a special character.

$$(26+26+10)(26+26+10+22)^6(26+26+10)$$

first and last characters have no special character.

choose lower case

c. *(2 marks)* How many different passwords are possible with the additional condition that the password must contain at least one upper case letter, one digit, one special character and the password must contain 8 distinct characters(lower case, upper case, digit, special character)?

$$\binom{26}{1}\binom{10}{1}\binom{22}{1}\binom{26+26+10+22-3}{5}8!$$

ways to arrange the 8 characters.

choose a digit

choose a special character

the characters must be distinct

d. *(2 marks)* If a password is selected at random what is the probability that the password is a rearrangement of the following eight letters Passwrds (the letter o is not included).

$$\frac{\frac{8!}{3!}}{(26+26+10+22)^8}$$

8! ← ways to arrange 8 characters.

3! ← remove repetition.

Question 4. At a bookclub meeting, 3 Marxist-Leninists, 5 Trotkists, 7 NDP members, and 11 Anarchists are present.

a. *(1 mark)* How many committees of 5 people can be formed?

$$\binom{3+5+7+11}{5}$$

b. *(2 marks)* How many committees of 5 people can be formed if the committee must include at least 1 Marxist-Leninist, 1 Trotkist, 1 NDP member, 1 Anarchist?

$$\binom{3}{1}\binom{5}{1}\binom{7}{1}\binom{11}{1}\binom{2+4+6+10}{1}$$

of 5 people

c. *(2 marks)* If a committee is selected at random, what is the probability that the committee includes Marxist-Leninist?

$$P(\text{No ML}) = \frac{\binom{3}{0}\binom{3+5+7+11-3}{5}}{\binom{3+5+7+11}{5}} = \frac{1\cdot 33649}{65780} = 51\%$$

d. *(3 marks)* Given that the meeting is in a room with 30 chairs arranged in a circle how many ways can the individual sit such that the Marxist-Leninist sit together with no empty chairs between them? note: 4 empty chairs

Group the ML together, so look at the arrangement of 28 including the empty chairs

arrangement of the ML

$$\frac{28!\,3!}{4!\,30}$$

← since in a circle shifting of chairs is the same permutation.

remove arrangement of empty chairs

Question 5. Consider the following fictional data: the percentage of students at Dawson College who use the following modes of transportation at least once a week: 11% walking (W), 10% cycling (B), 68% taking the metro (M), 2% walking and cycling, 4% cycling and walking, 3% walking and taking the metro, 1% walking, cycling and taking the metro.

a. *(2 marks)* If a Dawson student is selected at random what is the probability that the student walks, metros or cycles at least once a week.

$$P(W \cup B \cup M) = P(W) + P(B) + P(M) - P(W \cap B) - P(W \cap M) - P(B \cap M) + P(W \cap B \cap M)$$

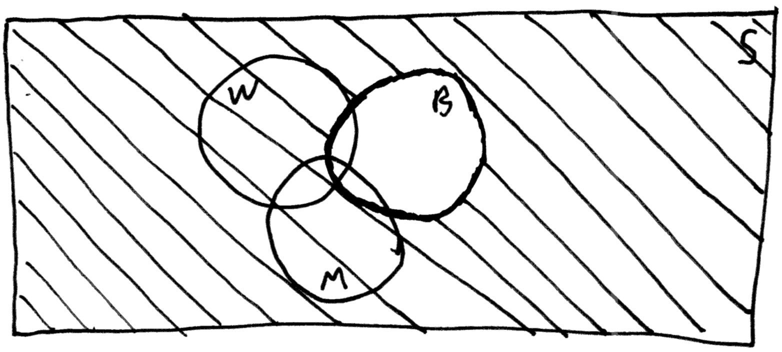
$$= 11\% + 10\% + 68\% - 2\% - 3\% - 4\% + 1\%$$

$$= 81\%$$

b. *(2 marks)* Describe and compute the probability of event B'. Draw a Venn diagram.

$P(B') = P(\text{does not bicycle at least once a week})$

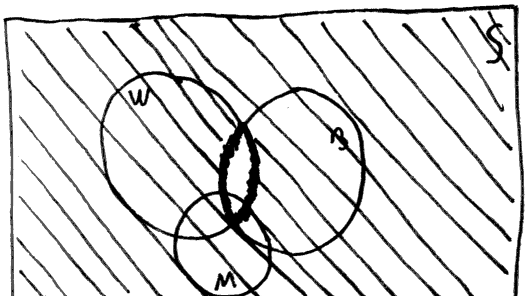
$= 1 - P(B) = 90\%$



B' is shaded \\\\

c. *(4 marks)* Describe and compute the probability of events $(W \cap B)'$ and $W' \cap B'$. Draw a Venn diagram for both cases.

$(W \cap B)' \equiv$ student who do not both walk and bicycle at least once a week

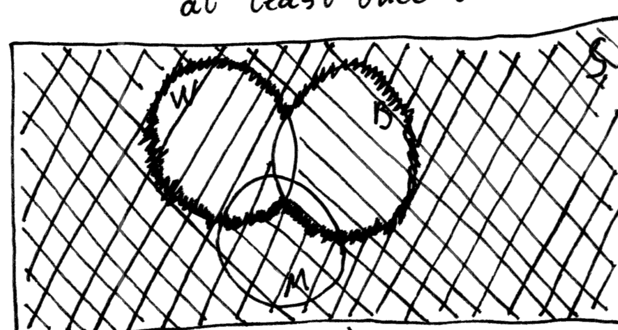


$P((W \cap B)') = 1 - P(W \cap B)$

$= 1 - 0.02$

$= 0.98$

$W' \cap B' \equiv$ student who do not walk and do not bicycle at least once a week.



W' is shaded \\\\\\

B' is shaded ///

So $W' \cap B' \equiv (W \cup B)'$

$P((W \cup B)') = 1 - P(W \cup B)$

$= 1 - (P(W) + P(B) - P(W \cap B))$

$= 1 - P(W) - P(B) + P(W \cap B)$

$= 1 - 0.11 - 0.10 + 0.02$

$= 0.81$

Bonus Question. *(3 marks)*

Prove: $P(A' \cap B') = 1 + P(A \cap B) - P(A) - P(B)$

As illustrated above 5c) $A' \cap B' = (A \cup B)'$ (called De Morgan Rule). So $P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - P(A) - P(B) + P(A \cap B)$