ASSIGNMENT #1

201-922-DW

SEPTEMBER 21, 2016

INSTRUCTOR: E. RICHER

SOLUTIONS

\# 2.2 (a) $\frac{18}{38}$ ← # red slots; ← # total slots

(b) $\frac{18}{38}$ ← # red slots; ← # TOTAL SLOTS

$= 0.4737$

(c) We are $\boxed{\text{NOT EQUALLY CONFIDENT}}$ BECAUSE after 300 spins we would expect THE PROPORTION OF "RED" spins SHOULD BE closer TO $(0.4737)(300) \approx 142$.

2.4 THE PROBABILITY OF ROLLING 2 SIXES is $\frac{1}{36}$ THE SAME is TRUE FOR 2 THREES.

So EACH series OF ROLLS is EQUALLY likely, even if 2 sixes are more valuable in THE GAME OF BACKGAMMON.

2.6 pair OF FAIR DICE

There are 36 OPTIONS OF POSSIBLE ROLLS

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

(a) GETTING A SUM OF 1 is impossible
THE SMALLEST POSSIBLE SUM is $1+1=2$

$P(\text{SUM}=1) = \boxed{0/36}$

(b) There are 4 WAYS TO GET A SUM OF 5:
(4,1) (3,2) (2,3) (1,4)

$P(\text{SUM}=5) = 4/36 = \boxed{1/9}$

(c) There is 1 WAY TO GET A SUM OF 12 : (6,6)

$P(\text{SUM}=12) = \boxed{1/36}$

2.10 (a) • There are $\underbrace{4}_{Q1} \times \underbrace{4}_{Q2} \times \underbrace{4}_{Q3} \times \underbrace{4}_{Q4} \times \underbrace{4}_{Q5} = 1024$

WAYS TO ANSWER THE EXAM

• There are $\underbrace{3}_{Q1} \times \underbrace{3}_{Q2} \times \underbrace{3}_{Q3} \times \underbrace{3}_{Q4} \times \underbrace{1}_{Q5} = 81$

WAYS TO HAVE ONLY THE 5th QUESTION RIGHT

SO $P(\text{ONLY Q5 RIGHT}) = \boxed{81/1024}$

(b) THERE IS ONLY 1 WAY TO GET ALL ANSWERS RIGHT

$P(\text{ALL RIGHT}) = \boxed{\frac{1}{1024}}$

(c) "AT LEAST 1 RIGHT" is 1 RIGHT OR
2 RIGHT OR
3 RIGHT OR
4 RIGHT OR
5 RIGHT

WE COMPUTE $P(\text{ALL WRONG}) = \frac{3 \times 3 \times 3 \times 3 \times 3}{1024} = \boxed{\frac{243}{1024}}$

"0 RIGHT" IS THE COMPLEMENT OF "AT LEAST 1 RIGHT"

SO

$P(\text{AT LEAST 1 RIGHT}) = 1 - \frac{243}{1024} = \boxed{\frac{781}{1024}}$

2.12

P (EXACTLY 1 DAY) = 0.25 (25%)
P (2 DAYS) = 0.15 (15%)
P (3 or more) = 0.28 (28%)

(a) P (0 DAYS) = 1 − (0.25 + 0.15 + 0.28)
= $\boxed{0.32}$ (32%)

(b) P (NO MORE THAN 1 DAY)

= P(0 DAYS) + P(1 DAY)
= 0.32 + 0.25
= $\boxed{0.57}$

(c) P (AT LEAST 1 DAY)

= P(1 DAY) + P(2 DAYS) + P(3 OR MORE DAYS)
= 0.25 + 0.15 + 0.28
= $\boxed{0.68}$

2.14

(a) P (EXCELLENT HEALTH ∩ DOESN'T HAVE COVERAGE)

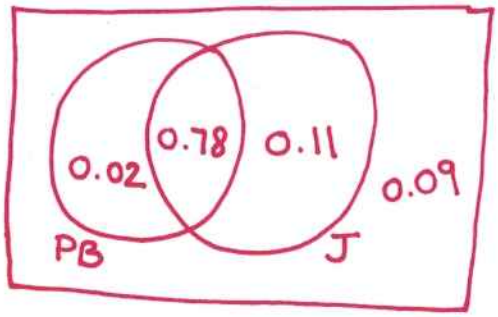
$$= \boxed{\frac{459}{20{,}000}}$$

(b) P (excellent HEALTH ∪ DOESN'T HAVE COVERAGE)

= P (EXC.) + P(NO COV.) − P (EXC. ∩ NO COV.)

$$= \frac{4657}{20000} + \frac{2524}{20000} - \frac{459}{20000} = \boxed{\frac{6722}{20000}}$$

#2.16



$$P(J \mid PB) = \frac{P(J \cap PB)}{P(PB)} = \frac{0.78}{0.80} = \boxed{0.975}$$

#2.20 Male ♂ Female ♀

(a) $P(♂\text{Blue} \cup ♀\text{Blue})$

$= P(♂\text{Blue}) + P(♀\text{blue}) - P(♂\text{Blue} \cap ♀\text{Blue})$

$= \frac{114}{204} + \frac{108}{204} - \frac{78}{204} = \boxed{\frac{144}{204}}$

(b) $P(♀\text{blue} \mid ♂\text{Blue}) = \frac{P(♀\text{blue} \cap ♂\text{blue})}{P(♂\text{blue})}$

$= \frac{78/204}{114/204} = \boxed{\frac{78}{114}}$

(c) $P(♀\text{blue} \mid ♂\text{Brown}) = \boxed{19/54}$

$P(♀\text{blue} \mid ♂\text{Green}) = \boxed{11/36}$

2.22

T_+ = TESTS POSITIVE

T_- = TESTS NEGATIVE

A_1 = HAS THROMBOSIS PREDISPOSITION

A_2 = DOES NOT HAVE THROMBOSIS PRED.

WHAT WE KNOW:

$P(A_1) = 0.03$

$P(A_2) = 0.97$

$P(T_+ | A_1) = 0.99$

$P(T_- | A_1) = 0.01$

$P(T_+ | A_2) = 0.02$

$P(T_- | A_2) = 0.98$

WE WANT TO FIND $P(A_1 | T_+)$:

$$P(A_1 | T_+) = \frac{P(A_1 \cap T_+)}{P(T_+)}$$

LET'S COMPUTE $P(T_+)$

THERE ARE TWO WAYS TO TEST POSITIVE

EITHER $T_+ \cap A_1$ OR $T_+ \cap A_2$

$$P(T_+ \cap A_1) = P(T_+ | A_1) \cdot P(A_1)$$
$$= (0.99) \cdot (0.03)$$
$$= 0.0297$$

$$P(T_+ \cap A_2) = P(T_+ | A_2) \cdot P(A_2)$$
$$= (0.02) \cdot (0.97)$$
$$= 0.0194$$

WE GET THE FOLLOWING

$$P(A_1 | T_+) = \frac{P(A_1 \cap T_+)}{P(T_+)}$$

$$= \frac{P(A_1 \cap T_+)}{P(T_+ \cap A_1) + P(T_+ \cap A_2)}$$

$$= \frac{0.0297}{0.0297 + 0.0194} = \boxed{0.605}$$

2.24

SW_{yes} = VOTED SCOTT WALKER

SW_{NO} = VOTED AGAINST SCOTT WALKER

C_{yes} = HAS college Degree

C_{NO} = HAS NO college Degree

WE KNOW:

$$P(SW_{yes}) = 0.53$$
$$P(SW_{No}) = 0.47$$

$$P(C_{yes} | SW_{yes}) = 0.37$$
$$P(C_{NO} | SW_{yes}) = 0.63$$
$$P(C_{yes} | SW_{NO}) = 0.44$$
$$P(C_{NO} | SW_{NO}) = 0.56$$

WE WANT TO FIND

$$P(SW_{yes} \mid C_{yes}) = ?$$

$$P(SW_{yes} \mid C_{yes}) = \frac{P(SW_{yes} \cap C_{yes})}{P(C_{yes})}$$

$$\begin{aligned} P(C_{yes}) &= P(C_{yes} \cap SW_{yes}) + P(C_{yes} \cap SW_{NO}) \\ &= P(C_{yes} \mid SW_{yes}) \cdot P(SW_{yes}) + P(C_{yes} \mid SW_{NO}) \cdot P(SW_{NO}) \\ &= (0.37)(0.53) + (0.44)(0.47) \\ &= 0.1961 + 0.2068 \\ &= 0.4029 \end{aligned}$$

SO $P(SW_{yes} \mid C_{yes}) = \dfrac{0.1961}{0.4029} = \boxed{0.487}$