

TEST 1

Introduction to Statistical Methods
(Analytical Chemistry)

201-922-DW

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Instructions:

- You have 75 minutes to complete the test
- No books, cell phones or other communication devices are permitted
- You must show all of your work in order to be credited with full marks
- Anyone suspected of cheating will be asked to leave

[QUESTION 1] (5 MARKS)

Consider the set $S = \{a, b, c, d, e\}$

(a) How many orderings of length 3 can be formed from the elements in S if elements cannot be repeated?

(b) How many orderings of length 3 can be formed from the elements in S if elements can be repeated?

(c) List all the of the permutations of length 2 that can be formed from the elements in S ?

$$(a) \quad {}_5P_3 = \frac{5!}{2!} = 5 \times 4 \times 3 = 60$$

$$(b) \quad 5 \times 5 \times 5 = 125$$

(c)

AB	BA	CA	DA	EA
AC	BC	CB	DB	EB
AD	BD	CD	DC	EC
AE	BE	CE	DE	ED

[QUESTION 2] (10 MARKS)

One-hundred and ninety-nine teachers were evaluated by their students, the table below summarizes the results of the evaluations:

		Teacher Evaluation					Total
		Excellent	Very good	Good	Fair	Poor	
Teachers' gender	Male	25	29	25	11	10	100
	Female	19	25	24	16	12	96
Total		44	54	49	27	22	196

(a) If we draw one of the evaluated teachers at random what is the probability that the teacher has an excellent evaluation and is male?

$$\frac{25}{196} = 0.128$$

(b) If we draw one of the evaluated *male* teachers at random, what is the probability that his evaluation is *good* or better?

$$\frac{25}{100} + \frac{29}{100} + \frac{25}{100} = \frac{79}{100} = 0.79$$

(c) If we draw one of the evaluated teachers at random, what is the probability that the teacher is *female* or that the teacher is evaluated as *excellent*?

$$\frac{96}{196} + \frac{44}{196} - \frac{19}{196} = \frac{121}{196} = 0.617$$

(d) If we draw from among the teachers who are evaluated as *poor* at random, what is the probability that the teacher is male?

$$\frac{10}{22} = 0.455$$

(e) Are the variables *male* gender and *good* teacher evaluation independent? Justify.

IF 'MALE' & 'GOOD' ARE INDEPENDENT THEN

$$P(\text{male} \cap \text{Good}) = P(\text{male}) P(\text{good})$$

WE KNOW FROM THE TABLE THAT $P(\text{male} \cap \text{Good}) = \frac{25}{196}$

WE ALSO KNOW THAT $P(\text{male}) = \frac{100}{196}$ & $P(\text{Good}) = \frac{49}{196}$

$$\text{So } P(\text{male}) \cdot P(\text{Good}) = \frac{100}{196} \cdot \frac{49}{196} = \frac{25}{196}$$

SINCE THE VALUES ARE THE SAME, THE VARIABLES 'MALE' & 'GOOD' ARE INDEPENDENT.

[QUESTION 3] (10 MARKS)

If you roll a pair of fair dice, what is the probability that

- (a) a sum of 8 is rolled?
- (b) a sum larger than 9 is rolled?
- (c) no even number appears on either die?
- (d) the same number appears on each die?
- (e) no six appear on either die?

$$(a) \quad P(\text{SUM}=8) = \frac{5}{36} \quad \left((4,4), (2,6), (6,2), (3,5), (5,3) \right)$$

$$(b) \quad P(\text{SUM}>9) = \frac{6}{36} \quad \left((5,5), (6,5), (5,6), (6,6), (6,4), (4,6) \right)$$

$$(c) \quad P(\text{NO EVEN \#}) = \frac{9}{36} \quad \left[\begin{array}{ccc} (1,1) & (3,1) & (5,1) \\ (1,3) & (3,5) & (5,3) \\ (1,5) & (3,3) & (5,5) \end{array} \right]$$

$$(d) \quad P(\text{NO 6}) = 1 - \frac{11}{36} \\ = \frac{25}{36}$$

WE LOOKED AT THE # OF WAYS OF HAVING A '6' ON EITHER DIE THE COMPUTED THE COMPLEMENT

(1,6) (6,1)
(2,6) (6,2)
(3,6) (6,3)
(4,6) (6,4)
(5,6) (6,5)
(6,6)

$$(e) \quad \frac{6}{36} \quad \left[\begin{array}{cc} (1,1) & (2,2) \\ (3,3) & (4,4) \\ (5,5) & (6,6) \end{array} \right]$$

[QUESTION 4] (5 MARKS)

If you flip a fair coin 9 times, what is the probability of

- (a) getting all heads?
- (b) getting exactly one heads?
- (c) getting at least one heads?

$$(a) \quad \left(\frac{1}{2}\right)^9 = \frac{1}{512}$$

$$(b) \quad 9 \times \left(\frac{1}{2}\right)^9 = \frac{9}{512}$$

$$(c) \quad 1 - \left(\frac{1}{2}\right)^9 = \frac{511}{512}$$

[QUESTION 5] (10 MARKS)

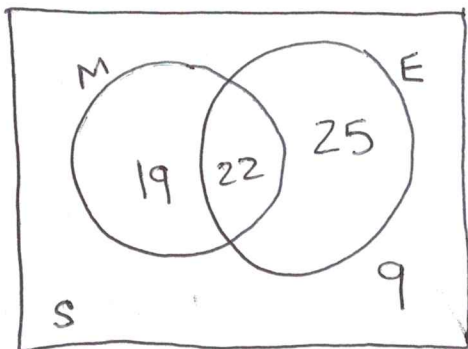
Seventy-five students were surveyed and asked whether they liked mathematics, english, both subjects or neither subject? The results reveal that 22 students like both subjects, 41 like mathematics and 9 students like neither.

Let M = student likes mathematics, E = student likes English

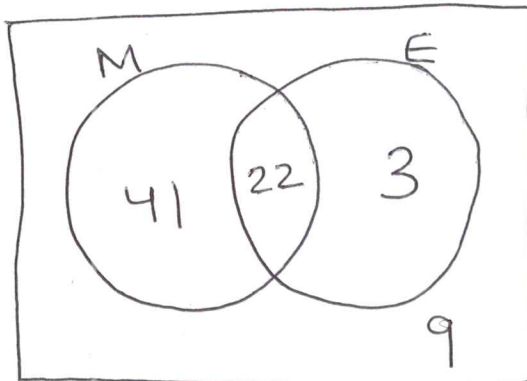
(a) Illustrate the situation using a Venn Diagram.

DUE TO THE AMBIGUOUS WORDING OF THE QUESTION, TWO SOLUTIONS WERE ACCEPTED

SOLUTION 1



SOLUTION 2



(b) Compute the following probabilities:

$$P(M) = \frac{41}{75}$$

OR SOLUTION 2

$$\frac{63}{75}$$

$$P(E) = \frac{47}{75}$$

$$\frac{25}{75}$$

$$P(M') = \frac{34}{75}$$

$$\frac{12}{75}$$

$$P(E' \cap M) = \frac{19}{75}$$

$$\frac{41}{75}$$

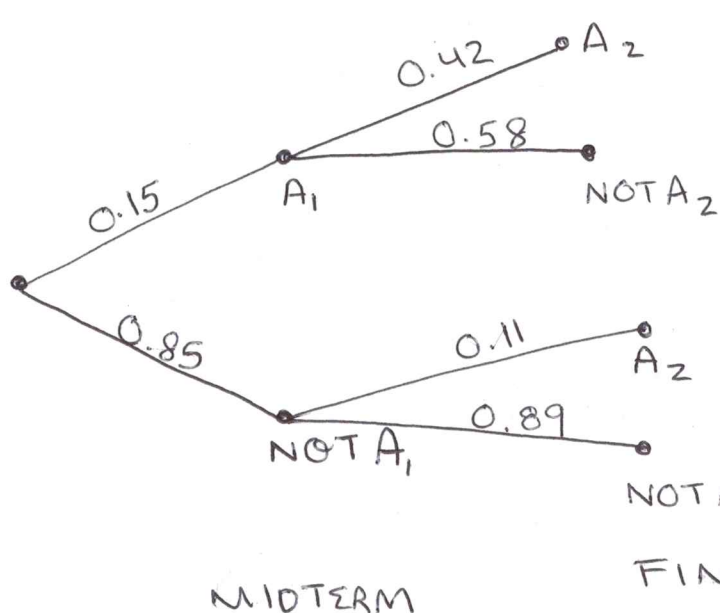
$$P(M|E) = \frac{22}{47}$$

$$\frac{22}{25}$$

[QUESTION 6] (5 MARKS)

Consider the midterm and final for a statistics class. Suppose 15% of students earned an A on the midterm. Of those students who earned an A on the midterm, 42% received an A on the final, and 11% of the students who earned lower than an A on the midterm received an A on the final.

You randomly pick a final exam and notice the student received an A. What is the probability that this student earned an A on the midterm?



$$P(A_1 \cap A_2) = (0.15)(0.42) = 0.063$$

$$P(A_1 \cap A_2^c) = (0.15)(0.58) = 0.087$$

$$P(A_1^c \cap A_2) = (0.85)(0.11) = 0.0935$$

$$P(A_1^c \cap A_2^c) = (0.85)(0.89) = 0.7565$$

$A_1 = A$ ON MIDTERM

$A_2 = A$ ON FINAL

WE WANT $P(A_1 | A_2)$

$$= \frac{P(A_1 \cap A_2)}{P(A_2)} = \frac{0.063}{0.063 + 0.0935} = \boxed{0.403}$$

[QUESTION 7] (5 MARKS)

Marbles are drawn from an urn, there are 5 red, 3 blue and 2 orange marbles in the urn.

(a) What is the probability of drawing a blue marble from the urn?

$$\frac{3}{10}$$

(b) If two marbles are drawn consecutively from the urn *with replacement* and the first marble drawn is blue, what is the probability of drawing a second blue marble?

$$\frac{3}{10}$$

(c) If two marbles are drawn consecutively from the urn *without replacement* and the first marble drawn is blue, what is the probability that the second marble is blue?

$$\frac{2}{9}$$

(d) [BONUS] (2 marks)

If two marbles are drawn consecutively from the urn (without replacement) what is the probability that the second marble drawn is blue?

THERE ARE 3 WAYS OF DRAWING A BLUE MARBLE SECOND:

ORANGE BLUE

$$P(OB) = \frac{2}{10} \cdot \frac{3}{9} = \frac{6}{90}$$

RED BLUE

$$P(RB) = \frac{5}{10} \cdot \frac{3}{9} = \frac{15}{90}$$

BLUE BLUE

$$P(BB) = \frac{3}{10} \cdot \frac{2}{9} = \frac{6}{90}$$

$$P(\text{Blue 2ND}) = \frac{6}{90} + \frac{15}{90} + \frac{6}{90} = \frac{27}{90} = \frac{3}{10}$$