

Student Name: _____

Student ID: _____

Comprehensive Examination (CE): Logic (Oral Examination)

Text: Proofs and Concepts: the fundamentals of abstract mathematics by Dave Witte Morris and Joy Morris.

<http://people.uleth.ca/~dave.morris/books/proofs+concepts.pdf>

Reading: Chapters 1 and 2.

Evaluation: The CE is associated to the Linear Algebra science course (201-NYC-05). Note that if the student fails the CE, the student cannot graduate. The CE mark for 201-NYC-05 will be either a pass or a fail. To pass the CE the student must obtain 60% or more on the oral examination.

Sample Oral Examination: See back of page.

Oral Examination Date and Location:

Consequence to Missing the Oral Examination Date: Failure of the CE unless a valid medical note is provided.

I hereby acknowledge that I have read, understand and agree to the terms of this Comprehensive Evaluation(CE).

Student Signature: _____

Date: _____

Sample Oral Examination:

Question 1. Given the following symbolization key:

A : Alexander Berkman loves Emma Goldman

B_1 : Alexander Berkman buys bread.

B_2 : Emma Goldman buys bread.

E : Emma Goldman loves Alexander Berkman.

F_1 : Alexander Berkman buys flowers.

F_2 : Emma Goldman buys flowers.

P_1 : Alexander Berkman protests.

P_2 : Emma Goldman protests.

Translate each English language statement into Propositional Logic.

- a. (1 mark) Emma buys flowers and Alexander buys bread if, neither Alexander loves Emma nor Emma loves Alexander.

Translate each Propositional Logic statement into English.

- b. (1 mark) $(\neg P_2 \wedge B_2) \iff E$

Question 2. (2 marks) Determine whether the following statement is a tautology, contradiction, or contingent statement. Justify your conclusion.

$$[(\neg A \rightarrow B) \wedge \neg B] \rightarrow A$$

Question 3. (2 marks) Determine whether the following is a valid argument. Justify your conclusion.

$$(\neg P_2 \wedge B_2) \iff E \therefore E$$

Question 4. (3 marks) Is the following possible? If it is possible, give an example. If it is not possible, explain why.

An invalid argument, the conclusion of which is a tautology.

Question 5.

- a. (0.5 mark) Translate the English statement into a propositional logic statement:
Emma Goldman does not love Alexander Berkman if Alexander does not buy flowers.
- b. (0.5 mark) Rewrite the propositional logic statement of part a. into a logically equivalent statement using the logical connective 'or'.
- c. (0.5 mark) Give the logical negation of the statement of part b. and distribute the negation using De Morgan Laws.
- d. (0.5 mark) Translate the propositional logic statement of part c. into an English statement.

Question 6. (2 marks) Using a truth table: determine whether the following two statements are logically equivalent. Justify.

$$\neg A \rightarrow B$$

and

$$(\neg A \rightarrow \neg B) \iff B$$

Question 7. (10 marks) Using only the rules of inference and the rules of replacement show that the following argument is valid using Fitch style natural deduction:

$$P \rightarrow Q, \neg P \rightarrow R, (Q \vee R) \rightarrow S, \therefore S$$

Rules of Inference

\rightarrow-elimination (or \rightarrowE) (or <i>Modus ponens</i>)	$\Phi \rightarrow \Psi, \Phi \therefore \Psi$
Modus tollens (or MT)	$\Phi \rightarrow \Psi, \neg \Psi \therefore \neg \Phi$
\leftrightarrow-introduction (or \leftrightarrowI) (or <i>Biconditional introduction</i>)	$\Phi \rightarrow \Psi, \Psi \rightarrow \Phi \therefore \Phi \leftrightarrow \Psi$
\leftrightarrow-elimination (or \leftrightarrowE) (or <i>Biconditional elimination</i>)	$\Phi \leftrightarrow \Psi \therefore \Phi \rightarrow \Psi$ and $\Phi \leftrightarrow \Psi \therefore \Psi \rightarrow \Phi$
\wedge-introduction (or \wedgeI) (or <i>Conjunction introduction</i>)	$\Phi, \Psi \therefore \Phi \wedge \Psi$
\wedge-elimination (or \wedgeE) (or <i>Simplification</i>)	$\Phi \wedge \Psi \therefore \Phi$ and $\Phi \wedge \Psi \therefore \Psi$
\vee-introduction (or \veeI) (or <i>Disjunction introduction, Addition</i>)	$\Phi \therefore \Phi \vee \Psi$
Disjunction elimination (or DE)	$\Phi \rightarrow \Psi, \Theta \rightarrow \Psi, \Phi \vee \Theta \therefore \Psi$
\vee-elimination (or \veeE) (or <i>Disjunctive syllogism</i>)	$\Phi \vee \Psi, \neg \Phi \therefore \Psi$
Hypothetical syllogism (or HS)	$\Phi \rightarrow \Psi, \Psi \rightarrow \Theta \therefore \Phi \rightarrow \Theta$
Constructive dilemma (or CD)	$\Phi \rightarrow \Psi, \Theta \rightarrow \Pi, \Phi \vee \Theta \therefore \Psi \vee \Pi$
Destructive dilemma (or DD)	$\Phi \rightarrow \Psi, \Theta \rightarrow \Pi, \neg \Psi \vee \neg \Pi \therefore \neg \Phi \vee \neg \Theta$
Absorption (or ABS)	$\Phi \rightarrow \Psi \therefore \Phi \rightarrow \Phi \wedge \Psi$

Rules of Replacement

Associativity (or Asso.)	$\Phi \square (\Psi \square \Theta) \equiv (\Phi \square \Psi) \square \Theta$ where $\square \in \{\wedge, \vee, \leftrightarrow\}$
Commutativity (or Comm.)	$\Phi \square \Psi \equiv \Psi \square \Phi$ where $\square \in \{\wedge, \vee, \leftrightarrow\}$
Distributivity (or Dist.)	$\Phi \wedge (\Psi \vee \Theta) \equiv (\Phi \wedge \Psi) \vee (\Phi \wedge \Theta)$ and $\Phi \vee (\Psi \wedge \Theta) \equiv (\Phi \vee \Psi) \wedge (\Phi \vee \Theta)$
Double negation (or DN)	$\neg \neg \Phi \equiv \Phi$
De Morgan's laws (or DM)	$\neg(\Phi \vee \Psi) \equiv \neg \Phi \wedge \neg \Psi$ and $\neg(\Phi \wedge \Psi) \equiv \neg \Phi \vee \neg \Psi$
Transposition (or Trans.)	$\Phi \rightarrow \Psi \equiv \neg \Psi \rightarrow \neg \Phi$
Material implication (or MI)	$\Phi \rightarrow \Psi \equiv \neg \Phi \vee \Psi$
Biconditional implication (or BI)	$\Phi \leftrightarrow \Psi \equiv (\Phi \rightarrow \Psi) \wedge (\Psi \rightarrow \Phi)$
Exportation (or Expo.)	$(\Phi \wedge \Psi) \rightarrow \Theta \equiv \Phi \rightarrow (\Psi \rightarrow \Theta)$
Tautology (or Taut.)	$\Phi \square \Phi \equiv \Phi$ where $\square \in \{\wedge, \vee\}$