Dawson College: **Linear Algebra**: 201-NYC-05-S04: Fall 2017

Name: ____________________

Test 1

This test is graded out of 38 marks. No books, notes, watches or cell phones are allowed. You are only permitted to use the Sharp EL-531XG or Sharp EL-531X calculator. Give the work in full; – unless otherwise stated, reduce each answer to its simplest, exact form; – and write and arrange your exercise in a legible and orderly manner. If you need more space for your answer use the back of the page.

Question 1. Given

$$M = \begin{bmatrix} 1 & 2 & 2 & -1 & 3 & 1 \\ 0 & 0 & 1 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where M is in row echelon form.

a. *(3 marks)* Find the reduced row echelon form of M.

b. *(2 marks)* Find 2 different row echelon form of M.

c. *(3 marks)* Find the solution set of the system of linear equations whose augmented matrix is M by using back substitution.

d. *(2 marks)* Find two particular solution of the system of linear equations whose augmented matrix is M.

e. *(2 marks)* Find the solution set of the homogeneous system of linear equations whose coefficient matrix is M.

f. *(2 marks)* Find a particular solution the homogeneous system of linear equations whose coefficient matrix is M when the solution of the first variable is equal to 1.

Question 2. Consider the matrices:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ 1 & -1 \end{bmatrix}, B = \begin{bmatrix} 2 & -5 & 2 \\ -3 & 2 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \\ 5 & 0 \end{bmatrix}, D = \begin{bmatrix} -1 & 1 \\ 4 & -3 \end{bmatrix}, E = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, F = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 0 & -5 & -1 \end{bmatrix}$$

Evaluate the following if possible, justify.

a. *(2 marks)* $\text{trace}(D)F^2$

b. *(2 marks)* $(F - AB)^T$

c. *(2 marks)* $\text{trace}(F)FB$

Question 3.*(2 marks)* Write the 3×2 matrix $B = [b_{ij}]$ whose entries satisfy

$$b_{ij} = \begin{cases} 1 & \text{if} \quad |i-j| \geq 1, \\ i+j & \text{if} \quad |i-j| < 1. \end{cases}$$

Question 4. *(3 marks)* Show that every square matrix can be written as a sum of two invertible matrices. *(Hint: Write A as a sum of lower and upper triangular matrice.)*

Question 4. *(3 marks)* Solve for A, if possible. A is a $\mathscr{M}_{2\times 2}$ diagonal matrix and satisfies $A^2 - 3A + 2I = 0$.

Question 5. Determine whether the following statements are true or false for any $n \times n$ matrices A and B. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. *(3 marks)* If $A^3 = 0$ then $A^2 - A + I$ is invertible. *(Hint: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$.)*

b. *(3 marks)* All elementary matrices are symmetric.

Question 6. *(3 marks)* Let A and B be $n \times n$ matrices.
Prove or disprove: If $A\mathbf{x} = \mathbf{0}$ and $B\mathbf{x} = \mathbf{0}$ have only the trivial solution then $(AB)\mathbf{x} = \mathbf{0}$ has only the trivial solution.

Question 7. Given the following system $\begin{cases} 2x & = a \\ 3x + 4y & = b \\ 5x + 7y + 11z & = c \end{cases}$

a. *(5 marks)* Find the inverse of the coefficient matrix.

b. *(2 marks)* Express the inverse of the coefficient matrix as a product of elementary matrices, if possible. *(Only the final answer will be graded.)*

c. *(2 marks)* For what value(s), if any, of a, b, c is the system consistent, justify.

d. *(2 marks)* Solve the system using the inverse for the value(s), if any, found in part b.

Bonus Question. *(3 marks)* Prove or disprove: If A is elementary then A^2 is elementary.