Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S2: Fall 2018

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Quiz 10

This quiz is graded out of 12 marks. No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1.

a. *(2 marks)* Find a homogeneous linear system of two equations in three unknowns whose solution space consists of those vectors in $\mathbb{R}^3$ that are orthogonal to $\mathbf{a} = (-3, 2, -1)$ and $\mathbf{b} = (0, -2, -2)$.

Let $\underline{x} = (x, y, z)$

$$\begin{cases} \underline{x} \cdot \underline{a} = 0 \\ \underline{x} \cdot \underline{b} = 0 \end{cases} \longrightarrow \begin{cases} -3x + 2y - z = 0 \\ \quad\;\; -2y - 2z = 0 \end{cases}$$

b. *(1 mark)* What kind of geometric object is the solution space?

The solution space is a line since the two equation are two non-parallel planes that intersect at a line.

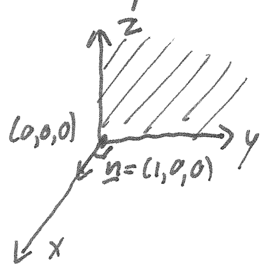
c. *(2 marks)* Find a general solution of the system obtained in part a.

$$\begin{bmatrix} -3 & 2 & -1 & 0 \\ 0 & -2 & -2 & 0 \end{bmatrix} \sim \xrightarrow{R_2 + R_1 \to R_1} \begin{bmatrix} -3 & 0 & -3 & 0 \\ 0 & -2 & -2 & 0 \end{bmatrix} \sim \begin{matrix} -\frac{1}{3}R_1 \to R_1 \\ -\frac{1}{2}R_2 \to R_2 \end{matrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Let $z = t$ then $x = -t$, $y = -t$ $\quad \therefore (x, y, z) = (-t, -t, t) = t(-1, -1, 1) \quad t \in \mathbb{R}$

Question 2.[1] *(4 marks)* Find the point of intersection of the line $(x, y, z) = (9, -1, 3) + t(5, 1, 1)$ where $t \in \mathbb{R}$ with the plane containing both the y-axis and the z-axis.

If the plane contains the y-axis and z-axis then it is orthogonal to the x-axis and contains the origin $\therefore$ $x = 0$ is the equation of the plane.



$(x, y, z) = (9 + 5t, -1 + t, 3 + t)$

Hence intersection with the plane when $x = 0$

$9 + 5t = 0$

$t = -9/5$

$\therefore$ point of intersection is $(x, y, z) = (9, -1, 3) - \frac{9}{5}(5, 1, 1)$

$= (0, -\frac{14}{5}, \frac{6}{5})$

Question 3. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

The general solution of the nonhomogeneous linear system $A\mathbf{x} = \mathbf{b}$ can be obtained by adding $\mathbf{b}$ to the general solution of the homogeneous linear system $A\mathbf{x} = \mathbf{0}$.

False, $\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\underline{x}} = \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{\underline{b}}$ its homogeneous system is $A\underline{x} = \underline{0}$ and only has the trivial solution since A is invertible.

i.e. $\underline{x} = \underline{0}$

But $\underline{x} = \underline{0} + \underline{b} = (1, 1)$ is not a solution of $A\underline{x} = \underline{b}$

[1] John Abbott Final Examination