Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S2: Fall 2018

Name: Y. Lamontagne

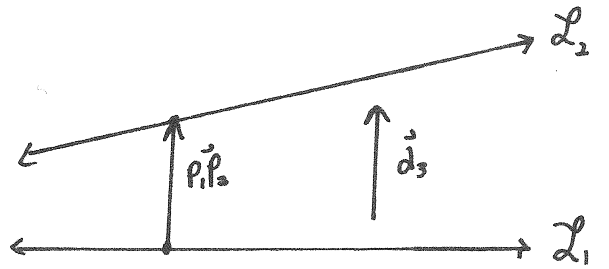
Quiz 11

This quiz is graded out of 12 marks. No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1.[1] *(5 marks)* Given the lines:

$$\begin{aligned}
\mathscr{L}_1 &: (x,y,z) = (1,2,-2) + t_1(1,2,1) \\
\mathscr{L}_2 &: (x,y,z) = (2,1,3) + t_2(1,2,3) \\
\mathscr{L}_3 &: (x,y,z) = (1,1,1) + t_3(2,7,3) \text{ where } t_1,\ t_2,\ t_3 \in \mathbb{R}.
\end{aligned}$$

which are all skew line to each other. Find the equation of the line which is parallel to $\mathscr{L}_3$ and which intersects both $\mathscr{L}_1$ and $\mathscr{L}_2$.



To find the equation of the line we can find the vector from $\mathcal{L}_1$ to $\mathcal{L}_2$ and parallel to $\vec{d}_3$.

$$\vec{P_1P_2} = (2+t_2,\ 1+2t_2,\ 3+3t_2) - (1+t_1,\ 2+2t_1,\ -2+t_1)$$
$$= (1+t_2-t_1,\ -1+2t_2-2t_1,\ 5+3t_2-t_1)$$

$$\vec{P_1P_2} = k\vec{d}_3$$

$$(1+t_2-t_1,\ -1+2t_2-2t_1,\ 5+3t_2-t_1) = k(2,7,3)$$

$$\left.\begin{aligned} 1+t_2-t_1 &= 2k \\ -1+2t_2-2t_1 &= 7k \\ 5+3t_2-t_1 &= 3k \end{aligned}\right\} \Rightarrow \begin{aligned} t_2-t_1-2k &= -1 \\ 2t_2-2t_1-7k &= 1 \\ 3t_2-t_1-3k &= -5 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -2 & -1 \\ 2 & -2 & -7 & 1 \\ 3 & -1 & -3 & -5 \end{array}\right]$$

$$\begin{array}{r} \sim -2R_1+R_2\to R_2 \\ -3R_1+R_3\to R_3 \end{array} \begin{bmatrix} 1 & -1 & -2 & -1 \\ 0 & 0 & -3 & 3 \\ 0 & 2 & 3 & -2 \end{bmatrix}$$

$$\sim R_2 \leftrightarrow R_3 \begin{bmatrix} 1 & -1 & -2 & -1 \\ 0 & 2 & 3 & -2 \\ 0 & 0 & -3 & 3 \end{bmatrix}$$

$$\sim -\tfrac{1}{3}R_3 \to R_3 \begin{bmatrix} 1 & -1 & -2 & -1 \\ 0 & 2 & 3 & -2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\begin{array}{r} 2R_3+R_1\to R_1 \\ \sim -3R_3+R_2\to R_2 \end{array} \begin{bmatrix} 1 & -1 & 0 & -3 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\sim \tfrac{1}{2}R_2 \to R_2 \begin{bmatrix} 1 & -1 & 0 & -3 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

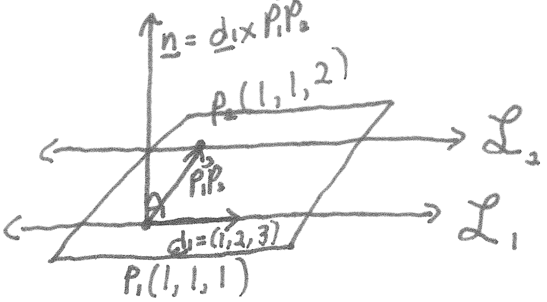
$$\sim R_2+R_1 \to R_1 \begin{bmatrix} 1 & 0 & 0 & -5/2 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$\therefore$ point on $\mathcal{L}_1$ when $t_1 = 1/2$

$$(x,y,z) = (1,2,-2) + \tfrac{1}{2}(1,2,1)$$
$$= \left(\tfrac{3}{2},\ 3,\ -\tfrac{3}{2}\right)$$

$$\therefore \mathcal{L}: (x,y,z) = \left(\tfrac{3}{2},\ 3,\ -\tfrac{3}{2}\right) + t(2,7,3)$$

Question 2. *(5 marks)* Give the equation of the described plane in general and parametric form.
Contains the lines $\vec{x} = (1,1,1) + t(1,2,3)$ and $\vec{x} = (1,1,2) + t(1,2,3)$.



$\vec{P_1P_2} = P_2 - P_1 = (1,1,2) - (1,1,1) = (0,0,1)$

$\underline{n} = \underline{d_1} \times \vec{P_1P_2} = \left(\begin{vmatrix} 2 & 0 \\ 3 & 1 \end{vmatrix}, -\begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix} \right)$

$\begin{matrix} 1 & 0 \\ 2 & 0 \\ 3 & 1 \end{matrix} = (2, -1, 0)$

$ax + by + cz = d$
$2x - y = d$
sub P_1 in above
$2(1) - 1 = d$
$1 = d$

$\therefore$ general equation of plane: $2x - y = 1$

$\therefore$ parametric equation of plane:
$\underline{x} = P_1 + s\vec{P_1P_2} + t\underline{d_1}$ where $s, t \in \mathbb{R}$
$= (1,1,1) + s(0,0,1) + t(1,2,3)$

Question 3. *(2 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

If $\vec{u}, \vec{v}$, and $\vec{w}$ are vectors in $\mathbb{R}^3$, where $\vec{u}$ is nonzero and $\vec{u} \times \vec{v} = \vec{u} \times \vec{w}$, then $\vec{v} = \vec{w}$.

False,
$\underline{u} = (1,1,1)$
$\underline{v} = (2,2,2)$
$\underline{w} = (3,3,3)$

$\underline{u} \times \underline{v} = (1,1,1) \times (2,2,2) = (1,1,1) \times (2(1,1,1)) = 2[(1,1,1) \times (1,1,1)] = 2\vec{0} = \vec{0}$

$\underline{u} \times \underline{w} = \quad \ldots \quad = \quad \ldots \quad = \quad \ldots \quad = 3\vec{0} = \vec{0}$

So $\vec{\underline{u}} \times \vec{\underline{v}} = \vec{\underline{u}} \times \vec{\underline{w}}$ but $\underline{v} \neq \underline{w}$