Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S2: Fall 2018

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Quiz 13

This quiz is graded out of 12 marks. No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

defintion:[1] The *union* of two sets A and B, denoted $A \cup B$, is the set of elements which are in A, in B, or in both A and B. In symbols, $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

The *intersection* of two sets A and B, denoted by $A \cap B$, is the set of all objects that are members of both the sets A and B. In symbols, $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

Question 1. True or False:

a. *(3 marks)* The intersection of any two subspaces of a vector space V is a subspace of V. Lets apply the subspace test.

Let $\underline{u}, \underline{v} \in A \cap B$. $\underline{u} + \underline{v} \in A \cap B$ since if $\underline{u}, \underline{v} \in A$ and $\underline{u}, \underline{v} \in B$ then $\underline{u} + \underline{v} \in A$ and $\underline{u} + \underline{v} \in B$ because both A and B are closed under addition (both subspaces)

Let $\underline{u} \in A \cap B$ and $r \in \mathbb{R}$. $r \cdot \underline{u} \in A \cap B$ since $\underline{u} \in A$ and $\underline{u} \in B$ then $r \cdot \underline{u} \in A$ and $r \cdot \underline{u} \in B$ because both A and B are closed under scalar multiplication (both subspaces)

$\therefore A \cap B$ is a subspace by the subspace test.

b. *(3 marks)* The union of any two subspaces of a vector space V is a subspace of V

Not a subspace. Suppose $A = \{(k,0) \mid k \in \mathbb{R}\}$ and $B = \{(0,k) \mid k \in \mathbb{R}\}$.
The above two sets can be shown to be subspaces of $\mathbb{R}^2$.
$(1,0) \in A$ and $(0,1) \in B$ but $(1,0) + (0,1) = (1,1) \notin A \cup B$ since $(1,1) \notin A$ and $(1,1) \notin B$.

Question 2. *(4 marks)* Determine whether the following polynomials span P_2: $p_1 = 1 - x + 2x^2$, $p_2 = 3 + x$, $p_3 = 5 - x + 4x^2$, $p_4 = -2 - 2x + 2x^2$.

Let $\underbrace{a + bx + cx^2}_{p(x)} \in P_2$, $p(x) = c_1 p_1(x) + c_2 p_2(x) + c_3 p_3(x) + c_4 p_4(x)$

$$a + bx + cx^2 = (c_1 + 3c_2 + 5c_3 - 2c_4) + (-c_1 + c_2 - c_3 - 2c_4)x + (2c_1 + 4c_3 + 2c_4)x^2$$

$$\left[\begin{array}{cccc|c} 1 & 3 & 5 & -2 & a \\ -1 & 1 & -1 & -2 & b \\ 2 & 0 & 4 & 2 & c \end{array}\right] \sim \begin{array}{l} R_1 + R_2 \to R_2 \\ -2R_1 + R_3 \to R_3 \end{array} \begin{bmatrix} 1 & 3 & 5 & -2 & a \\ 0 & 4 & 4 & -4 & b+a \\ 0 & -6 & -6 & 6 & c-2a \end{bmatrix} \sim \tfrac{1}{4}R_2 \to R_2 \begin{bmatrix} 1 & 3 & 5 & -2 & a \\ 0 & 1 & 1 & -1 & (b+a)/4 \\ 0 & -6 & -6 & +6 & c-2a \end{bmatrix}$$

$$\sim \; 6R_2 + R_3 \to R_3 \begin{bmatrix} 1 & 3 & 5 & -2 & a \\ 0 & 1 & 1 & -1 & (b+a)/4 \\ 0 & 0 & 0 & 0 & c-2a+6\frac{(b+a)}{4} \end{bmatrix}$$

$\therefore$ not consistent $\forall a, b, c$.

Not all $p(x)$ expressible as a lin. comb of $p_i(x)$

$\therefore \{p_i(x)\}$ does not span P_2

Question 3. *(3 marks)* Determine whether $\mathbb{R} \subseteq \text{span}(\{\sin^2 x, \cos^2 x\})$.

Let $x \in \mathbb{R}$ then we can express x as a linear combination of $\sin^2 x$ and $\cos^2 x$ since $1 = \sin^2 x + \cos^2 x$, so $x = x\sin^2 x + x\cos^2 x$. $\therefore x \in \text{span}(\{\sin^2 x, \cos^2 x\})$

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[1] from Wikipedia