Name: ____________________

Quiz 14

This quiz is graded out of 16 marks. No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1.[1] *(4 marks)* Given that $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ are three linearly independent vectors in $\mathbb{R}^n$. For which value(s) of k will the vectors $\mathbf{u}+2\mathbf{v}$, $\mathbf{v}+3\mathbf{w}$ and $k\mathbf{u}+\mathbf{w}$ be linearly dependent?

Question 2. Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. *(4 marks)* If $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are linearly dependent nonzero vectors, then at least one vector $\mathbf{v}_k$ is a unique linear combination of $\mathbf{v}_1, \ldots, \mathbf{v}_{k-1}$.

b. *(3 marks)* The set of 2×2 matrices that contain exactly two 1's and two 0's is a linearly independent set in $\mathscr{M}_{2\times 2}$.

[1] from a John Abbott Final Examination

Question 3.[2] *(1 mark each)*

I. Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be a linearly dependent set of vectors.
Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly dependent set of vectors.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly independent set of vectors.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ could be a linearly independent or linearly dependent set of vectors depending on the vectors chosen.
- D. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is a linearly independent set of vectors unless $\mathbf{u}_4$ is a linear combination of other vectors in the set.
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is a linearly independent set of vectors unless $\mathbf{u}_4 = \mathbf{0}$.
- F. none of the above

II. Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be a linearly independent set of vectors.
Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is also a linearly independent set of vectors unless $\mathbf{u}_4 = \mathbf{0}$.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ could be a linearly independent or linearly dependent set of vectors depending on the vectors chosen.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is also a linearly independent set of vectors unless $\mathbf{u}_4$ is a scalar multiple another vector in the set.
- D. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly independent set of vectors.
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly dependent set of vectors.
- F. none of the above

III. Let $\mathbf{u}_4$ be a linear combination of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.
Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is never a linearly dependent set of vectors.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly independent set of vectors.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is never a linearly independent set of vectors.
- D. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ could be a linearly dependent or linearly dependent set of vectors depending on the vector space chosen.
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a linearly dependent set of vectors unless one of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is the zero vector.
- F. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ could be a linearly dependent or linearly dependent set of vectors depending on the vectors chosen.
- G. none of the above

IV. Assume $\mathbf{u}_4$ is not a linear combination of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.
Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly dependent set of vectors.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is a linearly independent set of vectors unless one of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is the zero vector.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ could be a linearly independent or linearly dependent set of vectors depending on the vector space chosen.
- D. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is a linearly dependent set precisely when $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a linearly dependent set.
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is never a linearly dependent set of vectors.
- F. none of the above

V. Let $\mathbf{u} = \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix}$.
We want to determine by inspection (with minimal computation) if $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly dependent or independent.
Choose the best answer.

- A. The set is linearly independent because we only have two vectors and they are not scalar multiples of each other.
- B. The set is linearly dependent because two of the vectors are the same.
- C. The set is linearly dependent because one of the vectors is a scalar multiple of another vector.
- D. The set is linearly dependent because one of the vectors is the zero vector.
- E. The set is linearly dependent because the number of vectors in the set is greater than the dimension of the vector space.
- F. We cannot easily tell if the set is linearly dependent or linearly independent.

[2] from WeBWorK