Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S2: Fall 2018

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# Quiz 2

This quiz is graded out of 11 marks. No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** Given

$$M=\begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & \end{matrix} \\ \begin{bmatrix} 1 & 2 & 2 & -1 & 3 & 1 \\ 0 & 0 & 1 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \underset{-2R_3+R_2\to R_2}{\overset{-3R_3+R_1\to R_1}{\sim}} \begin{bmatrix} 1 & 2 & 2 & -1 & 0 & -2 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \overset{-2R_2+R_1\to R_1}{\sim} \underbrace{\begin{bmatrix} 1 & 2 & 0 & -3 & 0 & - \\ 0 & 0 & 1 & 1 & 0 & \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & \end{bmatrix}}_{R}$$

where $M$ is in row echelon form.

a. *(3 marks)* Find the reduced row echelon form of $M$.

b. *(1 mark)* Find two different row echelon form of $M$.

c. *(3 marks)* Find the solution set of the system of linear equations whose augmented matrix is $M$ by using back substitution.

d. *(1 mark)* Find two particular solution of the system of linear equations whose augmented matrix is $M$.

e. *(2 marks)* Find the solution set of the homogeneous system of linear equations whose coefficient matrix is $M$.

f. *(1 marks)* Find a particular solution the homogeneous system of linear equations whose coefficient matrix is $M$ when the solution of the first variable is equal to 1.

b) M and R

c) Let $x_2=s$, $x_4=t$ where $s,t\in\mathbb{R}$. Substitute in system

① $x_1+2s+2x_3-t+3x_5=$

② $x_3+t+2x_5=$

③ $x_5=1$

substitute ③ into ②

$x_3+t+2(1)=2$

④ $x_3=-t$

substitute ③, ④ into ①

$x_1+2s+2(-t)-t+3(1)=1$

$x_1=-2+3t-2s$

$\therefore (x_1,x_2,x_3,x_4,x_5)=(-2+3t-2s,\ s,\ -t,\ t,\ 1)$

$s,t\in\mathbb{R}$

d) $s=t=0$: $(x_1,x_2,x_3,x_4,x_5)=(-2,0,0,0,1)$

$s=1,\ t=0$:

$(x_1,x_2,x_3,x_4,x_5)=(-4,1,0,0,1)$

e) $[M\,|\,0]\sim\dots\sim \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & \end{matrix} \\ \begin{bmatrix} 1 & 2 & 0 & -3 & 0 & -2 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$

Let $x_2=s$, $x_4=t$, $x_6=r$ where $s,t,r\in\mathbb{R}$. Substitute in system

$$\begin{cases} x_1+2s-3t-2r=0 \\ x_3+t=0 \\ x_5+r=0 \end{cases}$$

$$\begin{cases} x_1=2r+3t-2s \\ x_3=-t \\ x_5=-r \end{cases}$$

$\therefore (x_1,x_2,x_3,x_4,x_5,x_6)=(2r+3t-2s,\ s,\ -t,\ t,\ -r,\ r)$ where $s,r,t\in\mathbb{R}$.

$1=x_1$

$1=2r+3t-2s$

Let $r=\frac{1}{2}$, $t=s=0$ then

$(x_1,x_2,x_3,x_4,x_5,x_6)=(1,0,0,0,-\frac{1}{2},\frac{1}{2})$