Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S2: Fall 2018

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Quiz 3

This quiz is graded out of 12 marks. No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(3 marks)* Prove that if the product CAC^T is defined, then the matrix A must be a square matrix.

Let C be an $m\times n$ matrix and A be a $p\times q$ matrix. Since CAC^T is defined then so is the product CA and AC^T. CA being defined implies that # col. of C = # rows of A. Hence $n=p$. AC^T being defined implies that # col of A = # rows of C^T. Hence $q=n$ since C^T is a $n\times m$ matrix.

$\therefore$ A is an $n\times n$ matrix.

Question 2. Determine whether the following statements are true or false for any $n \times n$ matrices A and B. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

1. *(3 marks)* If A and B are square matrices of the same order, then $\text{tr}(AB) = \text{tr}(A)\text{tr}(B)$.

False, if $A=B=I_2$ then $tr(AB) = tr(I_2I_2) = tr(I_2) = 1+1 = 2$

but $tr(A)tr(B) = tr(I_2)\,tr(I_2) = (1+1)(1+1) = 4$

As a result $tr(AB) \neq tr(A)tr(B)$

2. *(3 marks)* If A and B are square matrices of the same order, then $(AB)^T = A^TB^T$.

False, if $A=\begin{bmatrix}1&0\\2&1\end{bmatrix}$, $B=\begin{bmatrix}1&3\\0&1\end{bmatrix}$ then $(AB)^T = \left(\begin{bmatrix}1&3\\2&7\end{bmatrix}\right)^T = \begin{bmatrix}1&2\\3&7\end{bmatrix}$

but $A^TB^T = \begin{bmatrix}1&2\\0&1\end{bmatrix}\begin{bmatrix}1&0\\3&1\end{bmatrix} = \begin{bmatrix}7&2\\3&1\end{bmatrix}$

As a result $(AB)^T \neq A^TB^T$

3. *(3 marks)* If A is any matrix, then $\text{tr}(A^TA) \geq 0$.

True, let $A=[a_{ij}]_{n\times n}$ then $B = A^TA = [a_{ji}][a_{ij}] = B$,

$tr(A^TA) = tr(B) = b_{11} + b_{12} + \dots + b_{nn}$

$= \underbrace{a_{11}^2 + a_{21}^2 + \dots + a_{n1}^2}_{b_{11}} + \underbrace{a_{12}^2 + a_{22}^2 + \dots a_{n2}^2}_{b_{22}} + \dots + \underbrace{a_{1n}^2 + a_{2n}^2 + \dots + a_{nn}^2}_{b_{nn}}$

≥ 0 since the trace is the sum of squared terms.