Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S2: Fall 2018

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Quiz 4

This quiz is graded out of 12 marks. No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1.[1] *(3 marks)* Solve for the matrix X in the equation below:

$$(3XB)^{-1} + A = X^{-1}$$

Assume that all matrices involved are invertible.

$$\frac{1}{3}(XB)^{-1} - X^{-1} = -A$$

$$\frac{1}{3}B^{-1}X^{-1} - X^{-1} = -A$$

$$\left(\frac{1}{3}B^{-1} - I\right)X^{-1} = -A$$

$$(-A)^{-1}\left(\frac{1}{3}B^{-1} - I\right)X^{-1}X = (-A)^{-1}(-A)X$$

$$-A^{-1}\left(\frac{1}{3}B^{-1} - I\right)I = A^{-1}AX$$

$$-A^{-1}\left(\frac{1}{3}B^{-1} - I\right) = IX$$

$$-A^{-1}\left(\frac{1}{3}B^{-1} - I\right) = X$$

Question 2.[2] Let $A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ -1 & 2 \end{bmatrix}$.

a. *(3 marks)* Evaluate A^TA and find $(A^TA)^{-1}$.

$$A^TA = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 14 \end{bmatrix}$$

$$(A^TA)^{-1} = \frac{1}{2(14)-1(1)}\begin{bmatrix} 14 & -1 \\ -1 & 2 \end{bmatrix} = \frac{1}{27}\begin{bmatrix} 14 & -1 \\ -1 & 2 \end{bmatrix}$$

b. *(3 marks)* Evaluate AA^T and show that AA^T is not invertible.

$$AA^T = \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 10 & 3 & 5 \\ 3 & 1 & 2 \\ 5 & 2 & 5 \end{bmatrix} \sim_{R_1 \leftrightarrow R_3} \begin{bmatrix} 5 & 2 & 5 \\ 3 & 1 & 2 \\ 10 & 3 & 5 \end{bmatrix} \sim_{5R_2 \to R_2} \begin{bmatrix} 5 & 2 & 5 \\ 15 & 5 & 10 \\ 10 & 3 & 5 \end{bmatrix}$$

$$\sim \begin{array}{c} \\ -3R_1+R_2 \to R_2 \\ -2R_1+R_3 \to R_3 \end{array} \begin{bmatrix} 5 & 2 & 5 \\ 0 & -1 & -5 \\ 0 & -1 & -5 \end{bmatrix}$$

$$\sim \begin{array}{c} \\ \\ -R_2+R_3 \to R_3 \end{array} \begin{bmatrix} 5 & 2 & 5 \\ 0 & -1 & -5 \\ 0 & 0 & 0 \end{bmatrix} = R$$

The RREF of AA^T is not I since there is a row of zeros. after Hence by the equivalence thm A is not invertible.

Question 3. *(3 marks)* Prove: If A is an elementary matrix then A^2 is also an elementary matrix.

If A is an elementary matrix obtained by performing $R_i \leftrightarrow R_j$ on I then $A^2 = AA$ is equivalent to performing $R_i \leftrightarrow R_j$ on A. So $A^2 = I$ and I is an elementary matrix.

If A is an elementary matrix obtained by performing $cR_i \to R_i$ where $c \neq 0$ on I then $A^2 = AA$ is equivalent to performing $cR_i \to R_i$ on A. So A^2 is an elementary matrix which can be obtained by performing $c^2R_i \to R_i$ on I.

If A is an elementary matrix obtained by performing $cR_i + R_j \to R_j$ on I then $A^2 = AA$ is equivalent to performing $cR_i + R_j \to R_j$ on A. So A^2 is an elementary matrix which can be obtained by performing $2cR_i + R_j \to R_j$ on I.

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[1] From a John Abbott Final Examination

[2] From a John Abbott Final Examination