Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S2: Fall 2018

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Quiz 5

This quiz is graded out of 15 marks. No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(3 marks)* Prove that if A is an invertible matrix and B is row equivalent to A, then B is also invertible.

Given that B is row equivalent to A there exist E_i s.t. $E_k \cdots E_2 E_1 A = B$. Since all elementary matrices are invertible and A is invertible then B is invertible because it can be written as a product of invertible matrices.

Question 2. *(5 marks)* Let $Ax = 0$ be a homogeneous system of n linear equations in n unknowns, and let Q be an invertible $n \times n$ matrix. Prove that $Ax = 0$ has only the trivial solution if and only if $(QA)x = 0$ has only the trivial solution.

[$\Rightarrow$] premise: $Ax=0$ has only the trivial sol.

conclusion: $(QA)x=0$ has only the trivial sol.

By the equivalence thm. A is invertible since $Ax=0$ has only the trivial sol. QA is invertible since the product of invertible matrices is invertible.

$\therefore$ by equivalence thm. $(QA)x=0$ has only the trivial solution.

[$\Leftarrow$] premise: $(QA)x=0$ has only the trivial sol.

conclusion: $Ax=0$ has only the trivial sol.

By the equivalence thm. QA is invertible since $(QA)x=0$ only has the trivial sol. which implies that $\underbrace{(QA)^{-1}Q}A = I$

$CA = I$

where C is the inverse of A.

$\therefore$ A is invertible and $Ax=0$ has only the trivial sol. by the equivalence thm.

Question 3. *(3 marks)* We showed in class that the product of symmetric matrices is symmetric if and only if the matrices commute. Is the product of commuting skew-symmetric and symmetric matrices skew-symmetric? Explain.

The product is in fact skew symmetric. Suppose A is symmetric ($A^T = A$) and B is skew symmetric ($B^T = -B$) and both matrices commute. Then

$(AB)^T$

$= B^T A^T$

$= -BA$ since B is skew-symmetric and A is symmetric

$= -AB$ since A and B commute

so $(AB)^T = -AB$

$\therefore$ AB is skew symmetric.

Question 4. Determine whether the following statements are true or false for any $n \times n$ matrix A. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. *(2 marks)* If A^2 is a symmetric matrix, then A is a symmetric matrix.

False, Given $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ which is not symmetric but $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is symmetric

b. *(2 marks)* Elementary matrices are not row equivalent to the identity.

False, $I_2 \sim_{R_1 \leftrightarrow R_2} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = E$ E is an elementary matrix which is row equivalent to I.