Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S2: Fall 2018

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Quiz 6

This quiz is graded out of 13 marks. No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* Find the determinant of the matrix A by first using a cofactor expansion.

$$A = \begin{bmatrix} 2+\mathrm{tr}(A) & \det(A) & 1 \\ -2 & 3 & 1 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} -8 & \det(A) & 1 \\ -2 & 3 & 1 \\ 2 & 5 & 0 \end{bmatrix}$$

$$\mathrm{tr}(A) = 2 + 2\mathrm{tr}(A) + 3 + 0$$

$$-\mathrm{tr}(A) = 5$$

$$\mathrm{tr}(A) = -5$$

$$\det(A) = a_{13}C_{13} + a_{23}C_{23} + \overbrace{a_{33}C_{33}}^{0}$$

$$\det(A) = 1\begin{vmatrix} -2 & 3 \\ 2 & 5 \end{vmatrix} - 1\begin{vmatrix} -8 & \det(A) \\ 2 & 5 \end{vmatrix}$$

$$\det(A) = -2(5) - 3(2) - [-8(5) - 2\det(A)]$$

$$-\det(A) = 24$$

$$\det(A) = -24$$

Question 2.[1] Given $\det\left(\begin{bmatrix} a & b & c \\ d & e & f \\ 1 & 1 & 1 \end{bmatrix}\right) = 3$, compute the following determinants.

a. *(3 marks)* $\begin{vmatrix} 2 & 2 & 2 \\ a & b & c \\ d-3 & e-3 & f-3 \end{vmatrix}$

$$\overset{\frac{1}{2}R_1 \to R_1}{=} 2\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ d-3 & e-3 & f-3 \end{vmatrix} \overset{3R_1+R_3 \to R_3}{=} 2\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ d & e & f \end{vmatrix} \overset{R_1 \leftrightarrow R_2}{=} -2\begin{vmatrix} a & b & c \\ 1 & 1 & 1 \\ d & e & f \end{vmatrix}$$

$$\overset{R_2 \leftrightarrow R_3}{=} (-1)(-2)\begin{vmatrix} a & b & c \\ d & e & f \\ 1 & 1 & 1 \end{vmatrix}$$

$$= (-1)(-2)(3) = 6$$

b. *(3 marks)* $\begin{vmatrix} 0 & 0 & 5 & 10 \\ a & d & 2 & 5 \\ b & e & 2 & 5 \\ c & f & 2 & 5 \end{vmatrix}$

$$\overset{-2C_3+C_4 \to C_4}{=} \begin{vmatrix} 0 & 0 & 5 & 0 \\ a & d & 2 & 1 \\ b & e & 2 & 1 \\ c & f & 2 & 1 \end{vmatrix} \overset{C_3 \leftrightarrow C_4}{=} -\begin{vmatrix} 0 & 0 & 0 & 5 \\ a & d & 1 & 2 \\ b & e & 1 & 2 \\ c & f & 1 & 2 \end{vmatrix} \overset{-2C_3+C_4 \to C_4}{=} -\begin{vmatrix} 0 & 0 & 0 & 5 \\ a & d & 1 & 0 \\ b & e & 1 & 0 \\ c & f & 1 & 0 \end{vmatrix}$$

$$= 5C_{14} \quad \text{cofactor expansion along 1st row}$$

$$= 5(-1)^{1+4}\begin{vmatrix} a & d & 1 \\ b & e & 1 \\ c & f & 1 \end{vmatrix}$$

$$= -5\begin{vmatrix} a & b & c \\ d & e & f \\ 1 & 1 & 1 \end{vmatrix}$$

$$= -5(3) = -15$$

transpose does not change the det

Question 3. *(2 marks)* Determine whether the following statement is true or false for any $n \times n$ matrix A and B. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

$\det(A+B) = \det(A) + \det(B)$ False, Let $A = I_2$ and $B = -I_2$

$\det(A+B) = \det(0) = 0$ and $\det(A) + \det(B) = 1 + 1 = 2$

$\therefore \det(A+B) \neq \det(A) + \det(B)$

[1] From a John Abbott Final Examination