Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S2: Fall 2018

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Quiz 7

This quiz is graded out of 12 marks. No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* Consider two 4×4 matrices A and B, with $\det(A) = -2$ and $\det(B) = 3$. Find the determinant of M, knowing that $\det(2B^TMA^{-1}B) = \det(\text{adj}(A)A^2B)$. *Show every step!*

$$\det(2B^T)\det(M)\det(A^{-1})\det(B) = \det(\text{adj}(A))\det(A^2)\det(B)$$

$$2^4\det(B^T)\det(M)\frac{1}{\det(A)}\cancel{\det(B)} = (\det A)^{4-1}(\det(A))^2\cancel{\det(B)}$$

$$2^4\det(B)\det(M)\frac{1}{\det(A)} = (\det A)^5$$

$$\det(M) = \frac{(\det A)^6}{2^4\det(B)}$$

$$\det(M) = \frac{(-2)^6}{2^4(3)} = \frac{2^2}{3} = \frac{4}{3}$$

Question 2.[1] *(5 marks)* Let $\det\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right)$ be a nonzero value n. Use Cramer's Rule to solve for x_3 only in the system of linear equations below:

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & a & b & c \\ 0 & d & e & f \\ 0 & g & h & i \end{bmatrix}}_{A}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 3b+2c \\ 3e+2f \\ 3h+2i \end{bmatrix}$$

$$|A| = C_{11} = (-1)^{1+1}\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = n$$

$$|A_3| = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & a & 3b+2c & c \\ 0 & d & 3e+2f & f \\ 0 & g & 3h+2i & i \end{vmatrix} \overset{-2C_4 + C_3 \to C_3}{=} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & a & 3b & c \\ 0 & d & 3e & f \\ 0 & g & 3h & i \end{vmatrix}$$

$$\overset{\frac{1}{3}C_3 \to C_3}{=} 3\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & a & b & c \\ 0 & d & e & f \\ 0 & g & h & i \end{vmatrix} = 3C_{11} = 3(-1)^{1+1}\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 3n$$

$$x_3 = \frac{|A_3|}{|A|} = \frac{3n}{n} = 3$$

Question 3.[2] *(2 marks)* Determine whether the following statement is true or false for any $n \times n$ matrix A and B. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

If A is a skew-symmetric $n \times n$ matrix, i.e., $A^T = -A$, ~~show that~~ then when n is odd, $\det(A) = 0$.

True,

$A^T = -A$

$\det(A^T) = \det(-A)$

$\det(A) = (-1)^n\det(A$

$\det(A) = -\det(A)$ when n is odd

$2\det(A) = 0$

$\det(A) = 0$

[1]From a John Abbott Final Examination

[2]From a John Abbott Final Examination