Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S2: Fall 2018

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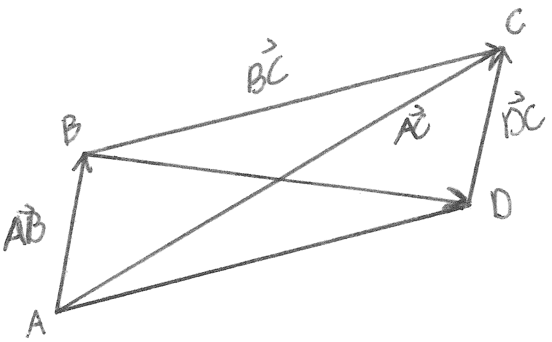
Quiz 8

This quiz is graded out of 12 marks. No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(4 marks)* Use vectors to prove that the two diagonals of a parallelogram bisect each other.



Need to show : $\frac{1}{2}\vec{AC} = \vec{AB} + \frac{1}{2}\vec{BD}$

Since a parallelogram $\vec{AB} = \vec{DC}$ *
$\vec{BC} = \vec{AD}$

$$
\begin{aligned}
LHS &= \tfrac{1}{2}\vec{AC} \\
&= \tfrac{1}{2}(\vec{AB} + \vec{BD} + \vec{DC}) \\
&= \tfrac{1}{2}(\vec{AB} + \vec{BD} + \vec{AB}) \quad \text{by } * \\
&= \tfrac{1}{2}(2\vec{AB} + \vec{BD}) \\
&= \vec{AB} + \tfrac{1}{2}\vec{BD}
\end{aligned}
$$

Question 2.[1] Let $\vec{u}$ and $\vec{v}$ be two unit vectors in $\mathbb{R}^n$ which are orthogonal to each other. Compute the following.

a. *(2 marks)* $(2\vec{u}+\vec{v})\cdot(\vec{u}-5\vec{v})$

$$
\begin{aligned}
&= (2\vec{u})\cdot\vec{u} - (2\vec{u})\cdot(5\vec{v}) + \vec{v}\cdot\vec{u} - \vec{v}\cdot(5\vec{v}) \\
&= 2(\vec{u}\cdot\vec{u}) - 10(\vec{u}\cdot\vec{v}) + \vec{v}\cdot\vec{u} - 5(\vec{v}\cdot\vec{v}) \\
&= 2\|\vec{u}\|^2 - 10(0) + 0 - 5\|\vec{v}\|^2 \\
&= 2(1)^2 - 5(1)^2 \\
&= -3
\end{aligned}
$$

b. *(2 marks)* $\|\vec{u}+4\vec{v}\|$

$$
\begin{aligned}
\|\vec{u}+4\vec{v}\|^2 = (\vec{u}+4\vec{v})\cdot(\vec{u}+4\vec{v}) &= \vec{u}\cdot\vec{u} + \vec{u}\cdot(4\vec{v}) + (4\vec{v})\cdot\vec{u} + (4\vec{v})\cdot(4\vec{v}) \\
&= \|\vec{u}\|^2 + 4(\vec{u}\cdot\vec{v}) + 4(\vec{v}\cdot\vec{u}) + 16(\vec{v}\cdot\vec{v}) \\
&= 1^2 + 4(0) + 4(0) + 16\|\vec{v}\|^2 \\
&= 1^2 + 16(1)^2 \\
&= 17
\end{aligned}
$$

$\therefore \|\vec{u}+4\vec{v}\| = \sqrt{17}$

Question 3. Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. *(2 marks)* If $\vec{u}\cdot\vec{v} = \vec{u}\cdot\vec{w}$, then $\vec{v} = \vec{w}$.

False, if $\vec{u} = (0,0)$ and $\vec{v} = (1,0)$, $\vec{w} = (0,1)$ then $\vec{u}\cdot\vec{v} = 0 = \vec{u}\cdot\vec{w}$ but $\vec{v} \neq \vec{w}$.

b. *(2 marks)* If $\vec{u}+\vec{v} = \vec{u}+\vec{w}$, then $\vec{v} = \vec{w}$.

True,

Let $\vec{u} = (u_1, u_2, \ldots, u_n)$, $\vec{v} = (v_1, v_2, \ldots, v_n)$ and $\vec{w} = (w_1, w_2, \ldots, w_n) \in \mathbb{R}^n$.

$\vec{u} + \vec{v} = \vec{u} + \vec{w}$

$(u_1+v_1, u_2+v_2, \ldots, u_n+v_n) = (u_1+w_1, u_2+w_2, \ldots, u_n+w_n)$,

So $u_i + v_i = u_i + w_i \quad \forall i$

$v_i = w_i \quad \forall i$

$\therefore \vec{v} = \vec{w}$

[1] From a John Abbott Final Examination