

Quiz 6

This quiz is graded out of 13 marks. No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) Find the determinant of the matrix A by first using a cofactor expansion.

$$A = \begin{bmatrix} 2 + \text{tr}(A) & \det(A) & 1 \\ -2 & 3 & 1 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} -8 & \det(A) & 1 \\ -2 & 3 & 1 \\ 2 & 5 & 0 \end{bmatrix}$$

$$\det(A) = a_{13}C_{13} + a_{23}C_{23} + a_{33}C_{33}$$

$$\det(A) = 1 \begin{vmatrix} -2 & 3 \\ 2 & 5 \end{vmatrix} - 1 \begin{vmatrix} -8 & \det(A) \\ 2 & 5 \end{vmatrix}$$

$$\det(A) = -2(5) - 3(2) - [-8(5) - 2\det(A)]$$

$$-\det(A) = 24$$

$$\det(A) = -24$$

$\text{tr}(A) = 2 + 2\text{tr}(A) + 3 + 0$
 $-\text{tr}(A) = 5$
 $\text{tr}(A) = -5$

Question 2.¹ Given $\det \begin{pmatrix} a & b & c \\ d & e & f \\ 1 & 1 & 1 \end{pmatrix} = 3$, compute the following determinants.

a. (3 marks) $\begin{vmatrix} 2 & 2 & 2 \\ a & b & c \\ d-3 & e-3 & f-3 \end{vmatrix} = \frac{1}{2}R_1 \rightarrow R_1 \quad 2 \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ d-3 & e-3 & f-3 \end{vmatrix} = 2 \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ d & e & f \end{vmatrix} = R_1 \leftrightarrow R_2 \quad -2 \begin{vmatrix} a & b & c \\ 1 & 1 & 1 \\ d & e & f \end{vmatrix}$

$$= R_2 \leftrightarrow R_3 \quad (1)(-2) \begin{vmatrix} a & b & c \\ d & e & f \\ 1 & 1 & 1 \end{vmatrix}$$

$$= (-1)(-2)(3) = 6$$

b. (3 marks) $\begin{vmatrix} 0 & 0 & 5 & 10 \\ a & d & 2 & 5 \\ b & e & 2 & 5 \\ c & f & 2 & 5 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 5 & 0 \\ a & d & 2 & 1 \\ b & e & 2 & 1 \\ c & f & 2 & 1 \end{vmatrix} = - \begin{vmatrix} 0 & 0 & 0 & 5 \\ a & d & 1 & 2 \\ b & e & 1 & 2 \\ c & f & 1 & 2 \end{vmatrix} = - \begin{vmatrix} 0 & 0 & 0 & 5 \\ a & d & 1 & 0 \\ b & e & 1 & 0 \\ c & f & 1 & 0 \end{vmatrix} = -5C_{44}$

cofactor expansion along 1st row

$$= -5(-1)^{1+4} \begin{vmatrix} a & d & 1 \\ b & e & 1 \\ c & f & 1 \end{vmatrix}$$

$$= +5 \begin{vmatrix} a & b & c \\ d & e & f \\ 1 & 1 & 1 \end{vmatrix}$$

$$= +5(3) = +15$$

transpose does not change the det

Question 3. (2 marks) Determine whether the following statement is true or false for any $n \times n$ matrix A and B . If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

$\det(A+B) = \det(A) + \det(B)$ False, Let $A = I_2$ and $B = -I_2$

$$\det(A+B) = \det(0) = 0 \quad \text{and} \quad \det(A) + \det(B) = 1 + 1 = 2$$

$$\therefore \det(A+B) \neq \det(A) + \det(B)$$

¹From a John Abbott Final Examination