

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Given $\mathcal{P}_1 : 2x + y - 3z = 6$, $\mathcal{P}_2 : -6x - 3y + 9z = -18$, $\mathcal{P}_3 : x + y + z = 1$, $\mathcal{L}_1 : \vec{x} = (1, 0, 1) + t(-4, -2, 6)$, $\mathcal{L}_2 : \vec{x} = (1, 1, 1) + s(2, 1, -3)$, and $\mathcal{L}_3 : \vec{x} = (1, 1, 1) + r(-2, -1, 4)$ where $s, t, r \in \mathbb{R}$.

a. (3 marks) Determine whether \mathcal{L}_1 and \mathcal{L}_2 are skew lines.

b. (3 marks) Find the parametric equation of the plane that contains \mathcal{L}_1 and \mathcal{L}_2 .

c. (3 marks) Find the intersection of \mathcal{L}_2 and \mathcal{P}_1 . What of significance can be said about the point of intersection and the point $(1, 1, 1)$.

d. (3 marks) Find the intersection of \mathcal{P}_1 , \mathcal{P}_2 and \mathcal{P}_3 .

e. (3 marks) Find the smallest angle between \mathcal{L}_3 and \mathcal{P}_3 .

f. (3 marks) Find the intersection of $2x + y - 3z = \alpha$, $-6x - 3y + 9z = \beta$, and $x + y + z = \gamma$ for fixed values of α , β , and γ given that intersection passes through the point $(\pi, \sqrt{2}, e)$.

Question 2. (*3 marks*) Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement. The general solution of the nonhomogeneous linear system $A\mathbf{x} = \mathbf{b}$ can be obtained by adding \mathbf{b} to the general solution of the homogeneous linear system $A\mathbf{x} = \mathbf{0}$.