Dawson College: Fall 2019: Linear Algebra (SCIENCE): 201-NYC-05-S5: **Quiz 10** name: Y. Lamontagne

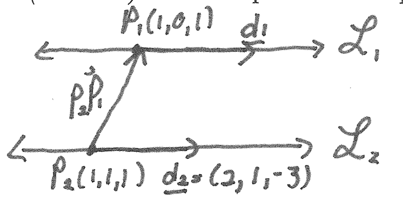
No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Given $\mathcal{P}_1 : 2x + y - 3z = 6$, $\mathcal{P}_2 : -6x - 3y + 9z = -18$ $\mathcal{P}_3 : x + y + z = 1$, $\mathcal{L}_1 : \vec{x} = (1, 0, 1) + t(-4, -2, 6)$, and $\mathcal{L}_2 : \vec{x} = (1, 1, 1) + s(2, 1, -3)$ where $s, t \in \mathbb{R}$.

a. *(3 marks)* Determine whether $\mathcal{L}_1$ and $\mathcal{L}_2$ are skew lines.

$\mathcal{L}_1$ and $\mathcal{L}_2$ are not skew lines since $\mathcal{L}_1 \parallel \mathcal{L}_2$ because $\underline{d}_1 = -2\underline{d}_2$

b. *(3 marks)* Find the parametric equation of the plane that contains $\mathcal{L}_1$ and $\mathcal{L}_2$.

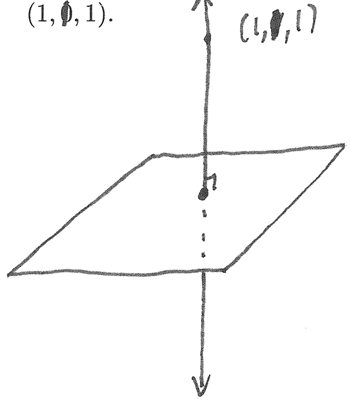


$\underline{x} = (1,1,1) + s\vec{P_2P_1} + t\underline{d}_2$

$= (1,1,1) + s(0,-1,0) + t(2,1,-3)$

$\vec{P_2P_1} = (1,0,1) - (1,1,1)$
$= (0,-1,0)$

c. *(3 marks)* Find the intersection of $\mathcal{L}_2$ and $\mathcal{P}_1$. What of significance can be said about the point of intersection and the point $(1, 1, 1)$.



Sub $x = 1+2s$
$y = 1+s$
$z = 1-3s$
into $2x+y-3z = 6$,

$2(1+2s) + (1+s) - 3(1-3s) = 6$

$2 + 4s + 1 + s - 3 + 9s = 6$

$14s = 6$

$s = \frac{6}{14} = \frac{3}{7}$

$\underline{x} = (1,1,1) + \frac{3}{7}(2,1,-3)$

$= \left(\frac{13}{7}, \frac{10}{7}, \frac{-2}{7}\right)$

is the closest point on the plane to $(1,1,1)$

d. *(3 marks)* Find the intersection of $\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathcal{P}_3$.

$$\begin{bmatrix} 2 & 1 & -3 & 6 \\ -6 & -3 & 9 & -18 \\ 1 & 1 & 1 & 1 \end{bmatrix} \sim_{3R_1+R_2 \to R_2} \begin{bmatrix} 2 & 1 & -3 & 6 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \sim_{R_1 \leftrightarrow R_3} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 2 & 1 & -3 & 6 \end{bmatrix}$$

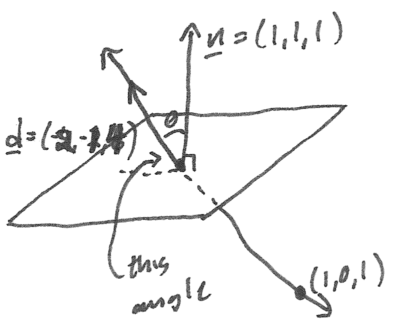
$$\sim_{R_3 \leftrightarrow R_2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & -3 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim_{-2R_1+R_2 \to R_2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -5 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim_{R_2+R_1 \to R_1} \begin{bmatrix} 1 & 0 & -4 & 5 \\ 0 & -1 & -5 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim_{-R_2 \to R_2} \begin{bmatrix} 1 & 0 & -4 & 5 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Let $z = t$ $x = 5+4t$
$y = -4-5t$

$(x,y,z) = (5+4t, -4-5t, t)$
$= (5,-4,0) + t(4,-5,1)$ $t \in \mathbb{R}$

e. *(3 marks)* Find the smallest angle between $\mathcal{L}_3$ and $\mathcal{P}_3$.



$\underline{d} \cdot \underline{n} = \|\underline{d}\| \|\underline{n}\| \cos\theta$

$1 = \sqrt{4+1+16}\sqrt{1+1+1} \cos\theta$

$1 = \sqrt{21}\sqrt{3} \cos\theta$

$\frac{1}{\sqrt{7}} = \cos\theta$

$\theta = \arccos\left(\frac{1}{\sqrt{7}}\right)$

$\therefore$ the angle is $\frac{\pi}{2} - \arccos\left(\frac{1}{\sqrt{7}}\right)$

f. *(3 marks)* Find the intersection of $2x + y - 3z = \alpha$, $-6x - 3y + 9z = \beta$, and $x + y + z = \gamma$ for fixed values of α, β, and γ given that intersection passes throught the point $(\pi, \sqrt{2}, e)$.

From d) and a theorem seen in class we know the sol. set of $2x+y-3z=0$, $-6x-3y+9z=0$, $x+y+z=0$ is $\underline{x} = t(4,-5,1)$ $\therefore$ by the same theorem the solution set is $(\pi, \sqrt{2}, e) + t(4,-5,1)$ where $t \in \mathbb{R}$.

Question 2. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement. The general solution of the nonhomogeneous linear system $A\mathbf{x} = \mathbf{b}$ can be obtained by adding $\mathbf{b}$ to the general solution of the homogeneous linear system $A\mathbf{x} = \mathbf{0}$.

False,

$\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\underset{\sim}{x}} = \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{b}$ has solution set $(0,1)$. But the solution set of $Ax = 0$ is $(0,0)$ since A is invertible (by the equivalence thm) and $\underset{\sim}{x} = b + (0,0) = (1,1)$ is not the solution set of $Ax = b$.